Wiggler and Undulator Magnets — A Review

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Wiggler and undulator magnets as radiation sources in electron storage rings are reviewed. The review covers

(a) the spectral properties of wiggler/undulator radiation,

(b) the technology available for their construction,

(c) the interaction of these devices with the stored electron beam,

(d) experience with existing devices,

(e) plans for future devices at laboratories around the world and

(f) beam line design considerations.

1. Introduction

It is clear that wiggler and undulator (w /u) magnets (periodic magnets which may be inserted into electron storage rings) produce synchrotron radiation with significantly higher brightness and/or broader spectral range than is possible with storage ring bending magnets. They are, therefore, increasingly in demand as the preferred radiation sources, particularly for the most difficult experiments seeking to observe small effects due to low sample concentrations or small reaction cross sections, or for experiments requiring high temporal or energy resolution. Future rings are likely to be built as all wiggler and undulator sources as has already been proposed.[1] In these rings the bending magnets serve to circulate the stored beam through a large number of wiggler / undulator magnets, with little or no use made of the radiation produced in the bending magnets.

In this paper we will review wiggler /undulator magnets as radiation sources in electron storage rings. We will cover (a) the spectral properties of wiggler /undulator radiation, (b) the technology available for their construction, (c) the interaction of these devices with the stored electron beam, (d) experience with existing devices, (e) plans for future devices at laboratories around the world and (f) beam line design considerations.

Theoretical and experimental work on wiggler / undulator magnets began about 30 years ago. Some of the early history can be found in other reviews[2,3] and will not be reported here.

We will restrict out attention to devices composed of alternating polarity transverse magnetic fields which cause the electron beam to undergo transverse oscillations resulting in radiation, but no net displacement or detection of the beam. Helical wiggler /undulator devices[4] (in which the transverse magnetic field is constant in magnitude but rotates in a helical fashion along the device) will not be considered. They require small apertures and hence their implementation in a storage ring poses special problems. One such device has been installed in the VEPP-2M storage ring in Novosibirsk (see section 5) where aperture requirements are small because a full energy damping injector is used. Helical undulators offer circularly polarized radiation.

To facilitate our discussion of wiggler /undulator magnets and, in particular to distinguish between wigglers and undulators, the parameter $K = 0.934\, B_o[\mathrm{T}]\lambda_u[\mathrm{cm}]$ is very useful. B_o is the peak magnetic field and λ_u (or λ_w) is the period length. When the magnetic field is sinusoidal $K = \gamma\delta$ where $\gamma = E_e/(m_o c^2)$ and 2δ is the full angular excursion of the electron beam traversing the wiggler / undulator magnet. For $K \leq 1$ the device is called an undulator. The angular excursion of the electron beam is less than or comparable to the natural opening angle of synchrotron radiation emission (γ^{-1}) and hence the radiation emerging from the device is concentrated in the smallest possible opening angle — i.e., it has the highest possible brightness. For an undulator with a large number of periods, interference effects in the radiation produced at a large number of essentially co-linear source points result in a spectrum with quasi-monochromatic peaks given by

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}\left[1 + \frac{K^2}{2} + \gamma^2\theta^2\right] n = 1,2,3,4,\ldots \quad (1)$$

where θ is the angle of observation relative to the average electron and direction and n is the harmonic number. For $K \ll 1$ only the fundamental peak ($n = 1$) is important. For $K \approx 1$ the power in the fundamental is a maximum and the first few harmonics have appreciable intensity.

The spectral width of the peaks is determined by the number of periods in the undulator, the transverse size and divergence of the electron beam and the angular acceptance of the detector. For an electron beam of negligible size and divergence and for a very small acceptance detector (i.e. a pinhole) the fractional bandwidth of the peaks is given by $\Delta\lambda/\lambda \approx 1/(nN)$, where N is the number of periods. In this case the on-axis brightness (photons/mm^2/mrad^2 within a constant bandwidth) increases as N^2.

For $K > 1$ the wavelength of the fundamental becomes longer [see equation (1)] and more harmonics appear. For $K >> 1$ the fundamental has very long wavelength and there are many closely spaced harmonics. In this limit the device is called a wiggler and the envelope of the spectrum approaches the familiar continuous synchrotron radiation spectrum characteristic of bending magnets. However, interference effects are still present in devices with $K >> 1$, especially at long wavelengths and in devices with many periods. At the shorter wavelengths, where these devices are more frequently used, interference effects, although still present, produce only very small variations about the normal continuous spectrum. For devices with $K >> 1$ the spectrum can be characterized by the critical energy calculated in the usual way; i.e.

$$\varepsilon_c\left[keV\right] = 0.665 B\left[T\right] E^2\left[GeV^2\right] \quad (2)$$

The intensity is enhanced due to the superposition of radiation from individual poles. Depending on the design of the wiggler and the emittance of the electron beam this enhancement factor can be as large as the number of poles in the wiggler. The beam line optics and the details of the spectral-angular distribution from an extended source must be considered in calculating the actual intensity delivered to an experimental station.

Devices with $1.5 \leq K \leq 5$ represent a transition region between wigglers and undulators. These are also being considered as radiation sources.[5]

High field wigglers offer extended spectral range compared with ring bending magnets. Extension of the spectral range can be achieved with a very simple 3 pole wiggler producing only one oscillation of the electron beam. Additional poles producing additional oscillations would provide higher intensity. As an example consider an 800 MeV ring. Typically such a ring would have bending magnet fields of about 1.2 T. This results in a spectrum with a critical energy of 511 eV and a maximum useful photon energy of 2-2.5 keV. A 5 T superconducting wiggler in the same ring produces a spectrum with a critical energy of 2.13 keV and a maximum useful energy reaching the important 8-10 keV region. Thus, a relatively low energy ring could provide radiation over an extremely broad spectral range. This has important implications for countries seeking a single moderate sized ring as a high intensity source in both the VUV and X-ray parts of the spectrum.

Recently rare-earth cobalt permanent magnet material has been used in the construction of wiggler / undulator magnets. This material has properties that are well suited for this purpose, making it possible to construct short period magnets with higher fields than can be obtained with normal electromagnets and, in some cases, also higher than can be obtained with superconducting magnets. Permanent magnet undulators have been built in Novosibirsk, SSRL and at the Photon Factory as VUV and soft X-ray ($h\nu \leq$ 1-2 keV) sources. The energy of the quasi-monochromatic peaks may be tuned by varying the gap.

Variable gap, in vacuum wiggler / undulator magnets and devices in which both the vacuum chamber and magnets have variable gap are now proposed[6] or under construction.[7] These devices open to provide the vertical aperture required for injection (typically 1.5-3 cm) and then close down to the smaller gaps (≤ 1 cm) that are sufficient for the stored beam. This small gap facilitates higher fields and shorter periods, increasing the brightness and spectral range of wigglers and extending undulator peaks to higher X-ray energies. For example, a permanent magnet undulator with a gap of 7.5 mm, period of 1.5 cm, peak field of 0.35 T ($K = 0.5$) is proposed[6] for SPEAR. At 3.5 GeV this device would produce a fundamental peak at 7 keV with far higher brightness than can be achieved with SPEAR bending magnets or wigglers, yet with much lower total radiated power.

It should be possible to modify existing rings or build new rings to make it possible to use undulators with gaps smaller than 7.5 mm, and correspondingly shorter periods. This would extend undulator high-brightness peaks to energies above 7 keV in existing rings and make it possible for even lower energy rings (≤ 2 GeV) to reach X-ray energies with small gap, short period devices. These possibilities are discussed further in section 4 of this paper and are also considered by Brown, Eisenberger and Winick.[8]

It is already clear that the limitation on the use of high-field, multipole wigglers in high current, multi-GeV storage rings will be determined by the ability to cool surfaces struck by the high power density of radiation that these devices can readily produce. The wigglers presently in use at SSRL and Novosibirsk radiate a total power of about 3 kW with 100 mA stored beams and produce a power density of about 200 W /mrad. Devices now in construction[7] will provide power densities a factor of 4 or more higher. The design of beam line elements (e.g., masks, mirrors, beryllium windows, crystals and gratings) that can function under this thermal load is becoming very complex.

These thermal problems make undulators appear increasingly attractive as radiation sources. The quasi-monochromatic nature of the spectrum produced by undulators means that they produce more useful photons per unit of radiated power than wigglers or bending magnets. For this reason we expect them to be ultimately the sources which can deliver the highest useful brightness.

2. Characteristics of the radiation from wiggler and undulator magnets

The spectral properties of synchrotron radiation emitted by relativistic charged particles has been treated in detail elsewhere.[9,10,11,12,13,14,15] In this section we wish to summarize the most important results, pertaining to realizable sources. In general, the number of photons radiated per unit solid angle per constant fractional bandwidth is given by the well-known formula,

$$\frac{dI}{d\Omega \frac{d\omega}{\omega}} = \frac{\alpha\omega^2}{4\pi^2}\left|\int_{-\infty}^{\infty} \mathbf{n}\times\mathbf{n}\times\beta e^{i\omega\left(\frac{t-\mathbf{n}\cdot\mathbf{r}}{c}\right)}dt\right|^2 \quad (3)$$

where $\boldsymbol{r}(t)$ is the particle trajectory, $c\boldsymbol{\beta} = c\, d\boldsymbol{r}/dt$, and $\boldsymbol{n}$ is the unit vector pointing from the particle to the observer. Ordinarily the observer is in the far-field region and $\boldsymbol{n}$ may be regarded as time independent.

For a particle moving in instantaneous circular motion with radius ρ (the bending magnet case) we have

$$\frac{dI}{d\Omega \frac{d\omega}{\omega}} = \frac{3\alpha}{4\pi^2}\gamma^2\left(\frac{\omega}{\omega_c}\right)\left(1+\theta^2\gamma^2\right)^2 \quad (4)$$

$$\mathbf{x}\left[K_{2/3}^2(\varepsilon)+\frac{\theta^2\gamma^2}{1+\theta^2\gamma^2}K_{1/3}^2(\varepsilon)\right]^2$$

where we have defined a critical frequency, $\omega_c = (3/2)\Upsilon^3 c/\rho$, $\varepsilon = (\omega/2\omega_c)(1 + \theta^2\Upsilon^2)^{3/2}$ and K_v is the modified Bessel function of order v. The first term, $K_{2/3}^2(\varepsilon)$, corresponds to radiation polarized in the plane of particle trajectory, and the second term corresponds to radiation normal to the plane. Thus, for observation points within the plane, the radiation is linearly polarized; outside the observation plane the radiation is in general elliptically polarized. It can also be shown from this formula that the bulk of the intensity falls, within a characteristic angle $1/\Upsilon$ and at frequencies $< 4\omega_c$. Thus, we see that synchrotron radiation emitted from a charged particle in uniform circular motion, is highly collimated, highly polarized, and extends into the X-ray spectral region for electrons with energy of order 2 GeV in magnetic fields of order 10 kG.

Since the radiation emitted by a particle in uniform circular motion is highly collimated in the vertical direction, it is useful to calculate the intensity integrated over vertical angles. The result is

$$\frac{dI}{d\theta_x \frac{d\omega}{\omega}} = \alpha\gamma f\left(\frac{\omega}{\omega_c}\right) \quad (5)$$

where

$$f(x) = \frac{\sqrt{3}}{2\pi}\int_x^{\infty} K_{5/3}(y)\,dy \quad (6)$$

2.1 Wiggler spectra

As discussed earlier a wiggler is a device with $K >> 1$. This is equivalent to defining it as a device with a periodically varying magnetic field that gives rise to a periodic trajectory with period λ_u and peak curvature ρ_0^{-1}, such that $\Upsilon\lambda_u >> 2\pi\rho_0$. For definiteness, we shall consider two examples of such device, the first being a sinusoidally varying magnetic field, and the second being a sequence of constant, but alternating polarity, magnetic fields. For the first example, the equations of motion are:

$$X''(z) = \rho^{-1}(z) = \rho_0^{-1}\cos\left(\frac{2\pi z}{\lambda_u}\right)$$

$$X'(z) = \frac{\lambda_u}{2\pi\rho_0}\sin\left(\frac{2\pi z}{\lambda_u}\right) \quad (7)$$

$$X(z) = -\left(\frac{\lambda_u^2}{4\pi^2\rho_0}\right)\cos\left(\frac{2\pi z}{\lambda_u}\right)$$

For the constant magnitude, alternating polarity wiggler, the equations are

$$X''(z) = \rho_0^{-1}\quad (j-1/4)\lambda_u \le z \le (j+1/4)\lambda_u$$
$$-\rho_0^{-1}\quad (j+1/4)\lambda_u \le z \le (j+3/4)\lambda_u \quad (8)$$

For the sinusoidal trajectory, we see that the amplitude of the angular excursion is $\lambda_u/2\pi\rho_0 = K/\Upsilon$ and that the corresponding angular spread for the constant-field device is $\lambda_u/4\rho_0$. This elementary result is fundamental to the design of wigglers as synchrotron radiation sources. For a given machine energy, one chooses ρ_0 to achieve the desired spectrum; one is then free to choose λ_u, either to emphasize the high intensity, low angular divergence limit ($\lambda_u \to 0$), or the opposing limit, where the source has lower on-axis intensity but can more easily illuminate simultaneous experimental stations. It must be remembered that in this limit, the oscillation amplitude of the particles may be large enough to seriously complicate the optical design. The implications on beam line design are further discussed in section 6.

The spectrum of radiation from a wiggler is essentially identical to the spectrum of a particle in uniform circular motion, provided that the instantaneous radius of curvature does not vary over the angular aperture the observer. It is a straightforward exercise to average equation (5) over the particle orbit, should this condition be violated. The effect of this averaging is shown in Figure 1 where the angle-integrated intensity is shown for a wiggler whose trajectory is given by equation (7), and also for a wiggler whose trajectory is given by equation (8), where the spectra are normalized to the same total radiated power. This normalization is accomplished by defining an effective critical frequency,

$$\omega_c^* = \frac{3}{2}\gamma^3\frac{c}{\rho^*}$$

where

$$\rho^* = \left\langle \rho^{-2} \right\rangle^{-1/2}$$

Two insertion devices with the same Υ, ρ^* and L will radiate the same total power, as by equation (23).

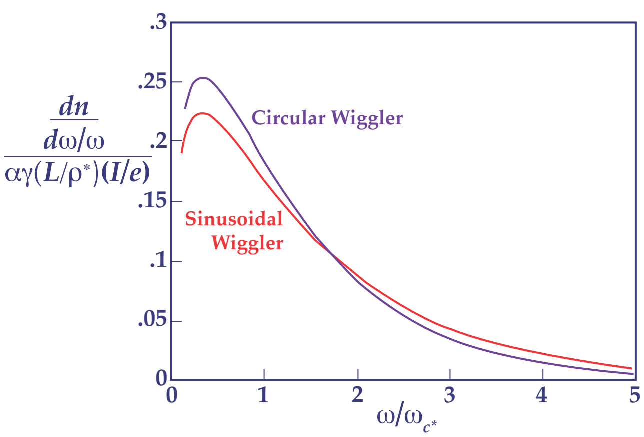

Figure 1 — Angle-integrated spectrum of synchrotron radiation from a wiggler magnet at fixed total power. a) Sinusoidal orbit. b) Sequence of circular arcs.

2.2. Undulator spectra: angle integrated

The spectrum of synchrotron radiation from a particle moving in a periodic trajectory has an additional characteristic frequency associated with it besides the critical frequency ω_c, namely, the doppler-shifted frequency of the particle's radiation emitted in a frame of reference moving uniformly with the same average velocity. This characteristic frequency, for a particle moving in a sinusoidally varying magnetic field, is given by

$$\omega_1 = \frac{\dfrac{4\pi\gamma^2 c}{\lambda_u}}{1 + \dfrac{K^2}{2} + \gamma^2\theta^2} \quad (9)$$

which is the same as equation (1), but expressed as a frequency rather than a wavelength. Strictly speaking, the spectrum observed at some polar angle θ will be composed only of multiples of the fundamental frequency ω_1, in the limit of large N. The angle-integrated intensity frequencies which are odd multiples of ω_1 is given by[8]

$$\frac{dI}{d\omega/\omega} = 4\pi\alpha N u\left[J_{(n+1)/2}(u) - J_{(n-1)/2}(u)\right]^2 \quad (10)$$

where N is the number of undulator periods and

$$u = n\left(\frac{K^2}{4}\right)\left(1+\frac{K^2}{2}\right); n = 1,3,5.... \quad (11)$$

Remarkably enough, the angle integrated spectrum is always peaked at $h\omega_1$ regardless of K. For $K \geq 3$ the spectrum at higher frequencies quickly blends into the constant curvature spectrum. It is also true that the intensity, integrated over all angles of the radiation at frequency ω_1 is always a significant factor higher than the intensity that one would have calculated under the assumption that one has a "bending magnet" spectrum of continuously varying curvature. This point is illustrated in Figure 2, which displays the angle integrated intensity from an undulator, as compared to a hypothetical "interference-free" wiggler radiating the same power, i.e. $K \to \infty$.

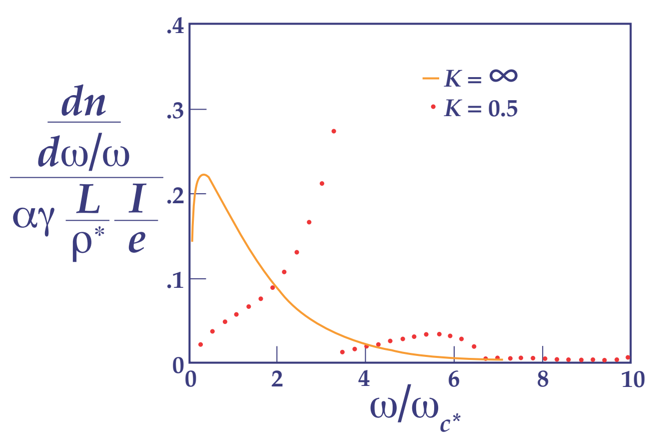

Figure 2a — Angle-integrated spectrum of synchrotron radiation from a sinusoidal undulator, $K = 0.5$ and $K = \infty$

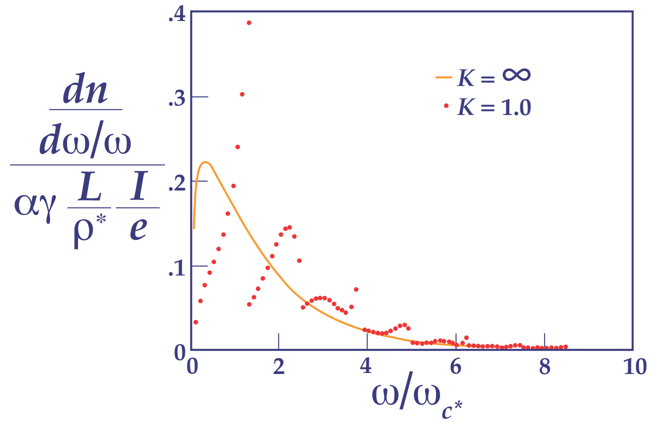

Figure 2b — Angle-integrated spectrum of synchrotron radiation from a sinusoidal undulator, $K = 1.0$ and $K = \infty$

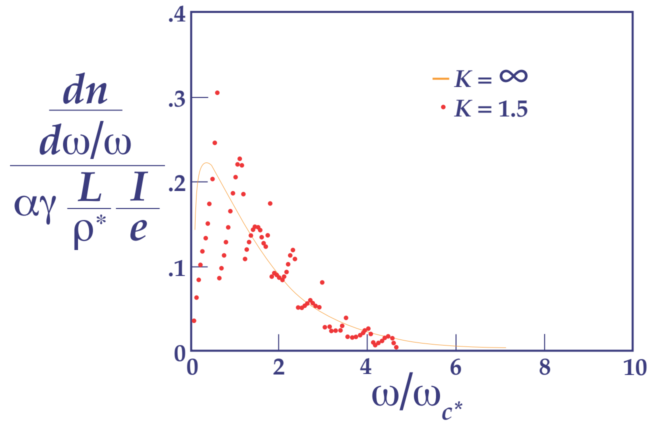

Figure 2c — Angle-integrated spectrum of synchrotron radiation from a sinusoidal undulator, K= 1.5 and K= ∞.

2.3 Undulator brightness

From the preceding arguments, one concludes that the "spectral rearrangement" of the radiation from an undulator significantly enhances the flux at multiples of the fundamental frequency, and that this enhancement is independent of the spatial and angular distribution of the electron beam, because it applies to the angle-integrated spectrum. However, if we consider the source brightness, we find an even larger increase over conventional sources, owing to the perfect (as $N \to \infty$) correlation between viewing angle and frequency, for a single electron.

For definiteness, we will take the electron beam distribution to be Gaussian in the transverse position and angular coordinates, with standard deviations σ_x, σ_y, σ_x', σ_y'. We will define the spectral brightness to be the number of photons per second per constant fractional bandwidth, and per unit area per steradian projected to the center of the source. For simplicity, we will neglect the length of the source, although this can easily be remedied by replacing σ_x by $[\sigma_x{}^2 + \sigma_{x'}{}^2(L/2)^2]^{1/2}$ etc.

It is then straightforward to calculate the brightness in the limit $N \to \infty$, from equation (10):

$$\frac{dI}{d\Omega dA \frac{d\omega}{\omega}} = \frac{1}{4\pi^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'}} \frac{dI}{\frac{d\omega}{\omega}} \quad (12)$$

In the opposite limit as $N \to 1$, it is also straightforward to show that:

$$\frac{dI}{d\Omega dA \frac{d\omega}{\omega}} = \frac{1}{2\pi^2 \sigma_x \sigma_y} \frac{N\gamma^2 n}{1 + \frac{K^2}{2}} \frac{dI}{\frac{d\omega}{\omega}} \quad (13)$$

where n = 1, 3, 5, ... Thus, the characteristic emission angle from a finite N device is of order

$$\gamma\theta \sim \frac{1 + K^2/2}{2Nn} \quad (14)$$

2.4 Polarization

A detailed treatment of the polarization of the radiation emitted by electrons passing through bending magnets, wigglers, and undulators is beyond the scope of this article. However, certain qualitative features can be easily described which apply to the majority of experimental situations.

For the bending magnet case, the radiation is completely plane polarized in the observation plane that is coplanar with the plane defined by the electron beam, as can be seen referring to equation (4). Away from the orbital plane, the radiation in general will have a vertical polarization component whose phase is directly correlated with the horizontal component, resulting in elliptically polarized light, with opposite helicity above and below the orbital plane. For the planar wiggler, whose radiation can be assumed to be an incoherent superposition of bending magnets of alternating curvature, the radiation is also plane polarized in the orbital plane. Away from the orbital plane, the sense of the helicity of photons emitted by successive poles is reversed, destroying the net helicity and resulting in a partially (linearly) polarized beam;

For radiation emitted by electrons passing through an undulator, the polarization properties depend upon the observation polar angle and azimuth, as well as upon the harmonic number. For the helical undulator[4] the on-axis radiation is circularly polarized, and for the planar undulator[12] the on-axis radiation is plane polarized. One might expect that the angular divergence of the electron beam would destroy this polarization. However, recalling that the on-axis radiation (for an infinite pole undulator) appears only at frequencies which are odd multiples of the fundamental frequency, a monochromator set at the fundamental frequency, for example, will select only the on-axis radiation, with a small contamination of second harmonic (red shifted to the fundamental frequency) at $\theta\Upsilon = [1 + K^2/2]^{1/2}$. This component is weak for small K, and can be significantly reduced with a suitable aperture.

3. Magnet design

Before starting to discuss the design of wiggler / undulators from the point of view of synchrotron light production, we want to summarize briefly the conditions that have to be satisfied to insure unperturbed operation of the storage ring. The first, and most obvious, condition is the requirement that the aperture is large enough both in the bend plane (usually horizontal) and perpendicular to it to accommodate the electron beam during injection, and enough of the tails of the beam when the injection and damping processes are finished so that the lifetime of the beam is not significantly reduced. Since the required horizontal aperture is usually much larger than the vertical aperture, and since the field produced on axis of a helical magnet goes down approximately like an exponential in the ratio of the largest aperture to the period length, it is unlikely that helical wiggler / undulator will be used extensively in the near future for the production of synchrotron radiation. We will, therefore, not discuss the design of helical magnets.

Another obvious requirement is that the wiggler / undulator should not introduce a significant net lateral displacement of the central beam trajectory, nor a net change in direction of this trajectory. In the vertical plane this is accomplished by assuring that the fields produced by the wiggler / undulator have horizontal midplane symmetry. In the horizontal plane, these conditions are customarily, and best, satisfied in the following manner: The wiggler / undulator is designed such that at its longitudinal midpoint the field goes through a maximum and has symmetry about the transverse midplane passing through this point. At each end of the wiggler / undulator a "half pole" is used with strength that is adjusted such that the field component perpendicular to the midplane, integrated along the beam direction from the longitudinal midpoint to well outside the magnet (in each direction), equals zero.

Lastly, the wiggler / undulator should modify the optics of the storage ring only to the extent that corrections can be made in the lattice. This is discussed more fully in the next section.

Wigglersy / undulators can be classified by the technology used to generate the magnetic fields: superconductivity, conventional coil and steel magnets, and permanent magnets; and we discuss them in this order.

Superconducting magnets are the only magnets that can produce fields that significantly exceed 2 T peak field. High field wigglers have usually only a few poles and have been built for midplane fields up to 6 T. In the future it should be possible to build superconducting wigglers with fields up to 10 T. Superconducting undulators[16] (with many poles and fairly low fields) have been built, but it is now probably easier to equal or exceed their performance with the permanent magnet technology discussed below.

Conventional wigglers / undulators can be designed to reach midplane fields up to about 2 T. Their design and construction is fairly straightforward and their use quite convenient. Conventional wigglers / undulators are usually designed to have a field distribution that is flatter than a sinusoidal distribution near the peaks of the field. This has obviously an effect on both the synchrotron radiation production and the beam-optical properties of the wiggler / undulator. A property of conventional wiggler / undulators that can make them unattractive is their energy consumption: power dissipation of the order of 200-500 kW is not uncommon and clearly not desirable in view of the high cost of energy.

Permanent magnet wiggler / undulators are relatively new devices which have great potential, and we discuss them therefore in more detail. The reason to go to this technology is fairly simple: One is generally highly motivated to make wiggler / undulator with short period length. In the case of the undulator, decrease of the period length with constant K and constant length of the undulator leads to a brighter source of higher energy photons. Similarly, the intensity of the radiation in the forward direction of a wiggler is increased with the number of poles, which is inversely proportional to the period length in a wiggler of given overall length. If one decreases the linear dimensions of a superconducting or conventional magnet while keeping the field strength fixed (or increasing it, as is necessary if K is frozen), then the current density has to go up. In the part of the parameter space that is of interest today, conventional wiggler / undulators are in a region where large current densities are required, posing severe problems which severely limit the achievable combination of fields and linear dimensions. Permanent magnets do not suffer from this limitation because the peak field produced in the gap of a permanent magnet stays constant with the scaling of all linear dimensions.

Rare-earth cobalt permanent magnet materials (e.g., $SmCo_5$) have certain unusual properties that make them well suited for wiggler / undulator applications. Magnetically they are highly anisotropic, with an easy axis along which the B vs $\mu_o H$ curve is a straight line in the 1st, 2nd and part of the 3rd quadrants with a slope close to unity. The remanent field, B_r, (the intersection of that line with the B axis) is typically 0.8-1 T. In the direction perpendicular to the easy axis the B vs $\mu_o H$ curve is a straight line through the origin with a slope that is also close to unity. As a result of these properties the material behaves very nearly like a vacuum with an impressed current. In a device in which all of the magnetic material has these properties (e.g., no steel is used) it is possible to predict analytically the field that will result almost any configuration of blocks.

For example, the simplest conceptual design samarium cobalt wiggler / undulator is shown in Figure 3. The, first of this type, an undulator, was built at the Berkeley Laboratory for use at SSRL and has described.[17,18] The gap field is given by.[19]

$$B_z - iB_y = 2B_r \sum_{u=0}^{\infty} \cos\left[mk(z+iy)\right] e^{-mkg/2} G_m$$

$$G_m = \frac{\sin\left(\frac{m\varepsilon\pi}{M'}\right)}{\frac{m\pi}{M'}}\left(1-e^{-mkh}\right); m = 1+uM' \qquad (15)$$

In these equations B_r is the remanent field, $k = 2\pi/\lambda_u$, λ_u is the period length, h is the height of the blocks, g is the full gap and M' is the number of blocks per period in each half (i.e., upper or lower) of the device. The equations are valid when the width of the blocks (in the x direction) is much greater than the gap. Finite width effects can also be expressed analytically and formulas and graphs are given in reference 19.

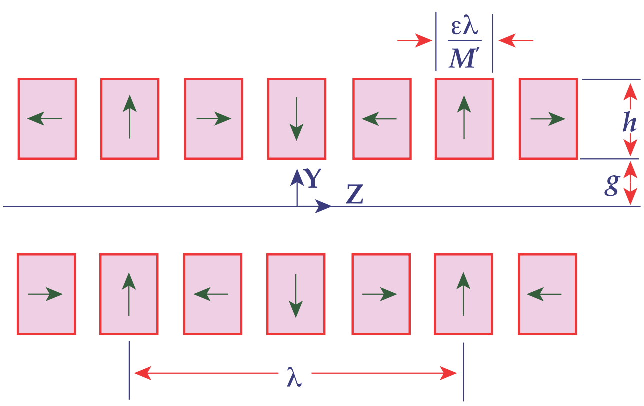

Figure 3 — Vertical cross section of a pure rare-earth cobalt wiggler / undulator with four blocks per period (conceptual). The electron beam moves in the z direction. The arrows indicate the direction of magnetization.

For reasonable choices of M' and h, G_1 is of the order of 0.7-0.85 leading to a rather strong wiggler / undulator. As is clear from equation (1), the field produced by this type of wiggler / undulator is nearly purely sinusoidal. The first harmonic ($m = 1 + M'$) can be made zero, if desired, by choosing $\varepsilon = 1 - 1/(M' + 1)$. For many purposes the field of the device calf be simply considered to be a sine wave with period and amplitude

$$B_0 = 2B_r e^{\frac{-\pi g}{\lambda_u}} \frac{\sin\left(\frac{\pi}{M'}\right)}{\frac{\pi}{M'}}\left[1-e^{\frac{-2\pi h}{\lambda_u}}\right] \qquad (16)$$

It is most convenient to build such a device with four blocks per period ($M' = 4$) and with blocks of square cross section ($h = \lambda_u/4$), In this case, for $B_r = 0.9$ T (which is readily available) equation (16) reduces to $B_o[\mathrm{T}] = 1.28e^{-\pi g/\lambda_u}$. Higher fields can be obtained with more blocks per period but this requires that blocks be magnetized along directions other than along edges. Rectangular blocks also give higher field. For example for $h = \lambda_u/2$, $M' = 4$ and $B_r = 0.9$ T equation (16) becomes $B_o[\mathrm{T}] = 1.55e^{-\pi g/\lambda_u}$.

Using this type of undulator in free electron laser experiments with fairly low energy electron beams, some workers have experienced problems stemming from poor quality control by the manufacturer over strength and direction of the magnetization within the samarium-cobalt blocks.[20] However, remedies have been found to correct this problem.[21]

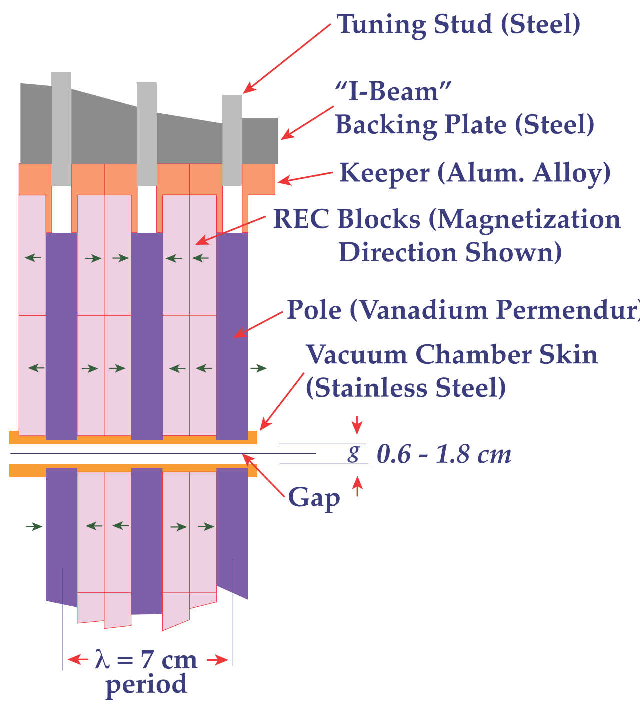

Figure 4 — Vertical cross section of a hybrid wiggler / undulator using rare-earth cobalt (REC) material plus steel poles.

For the stiffer beams encountered in storage rings, these problems are not nearly as severe. If the total dipole moment of the individual blocks spreads only over a few percent, sorting of the blocks according to strength and appropriate allocation within the wiggler / undulator will in general be sufficient.[17]

In a hybrid design that uses steel in addition to samarium cobalt, the peak field can be increased significantly over that produced by the pure samarium cobalt wiggler / undulator. Figure 4 shows the arrangement of steel and samarium cobalt in the upper half of such a magnet. Design details of a specific wiggler will be presented at this conference,[7] and the general design philosophy and details will be discussed elsewhere.[22]

We give here only the performance limit of this type of magnet. The achievable peak field may be expressed by

$$B_0\left[T\right] = 3.33\exp\left[-\frac{g}{\lambda_u}\left(5.47 - 1.8\frac{g}{\lambda_u}\right)\right] \quad (17)$$

This equation assumes the use of high permeability steel (vanadium permendur) poles and samarium cobalt material with a remanent field of 0.9 T. It fits rather well with the results of a sequence of computer runs for $0.07 \le g/\lambda_u \le 0.7$ giving a maximum fields of about 2.3 T.

Compared to the pure samarium cobalt wiggler / undulator, this hybrid has significant harmonics. The fractional third harmonic increases monotonically with the peak field and reaches approximately 28% at B = 2.3 T. Its sign is such as to sharpen the peak. The fifth harmonic behaves similarly and is about 7% at the peak field. Both these harmonics can be controlled to some extent without losing significantly in peak field level. Higher harmonics are not present in any significant amount.

The peak field of a hybrid wiggler / undulator is less sensitive to variations in material properties than the pure samarium cobalt device. Also, the hybrid design lends itself to the incorporation of steel tuning studs[7] to trim the peak value of the field at each pole (see Figure 4). Thus, it is possible to achieve very high field uniformity along the length of the magnet. These features plus its improved performance make it superior to the pure samarium cobalt design. It is, however, more difficult to design hybrids because, unlike the pure samarium cobalt wiggler / undulator which can be analyzed {fairly readily analytically (including three dimensional effects), the hybrid analysis is more complex requiring a computer and even some model measurements.

Field variation in a permanent magnet wiggler / undulator is most easily achieved by changing the gap. Experience with the undulator described in reference 17 indicates that the gap can be varied at will by the experimenter with no adverse effects on the operation of the storage ring or experiments on other beam lines.

4. Effects on the stored beam

Wigglers and undulators have been installed at a number of storage rings and are now operating routinely. Their effects on the stability and quality of the stored beam have been calculated[3,23,24,25] and evaluated experimentally at several of these installations.[18,26,27,28]

The form of the magnetic field in a wiggler / undulator modifies the magnetic lattice of the storage ring in several ways and produces several effects on the stored beam. Some of these effects are readily calculated and easily compensated while others are less obvious and depend on fine detail of the wiggler / undulator in question and of the storage ring lattice.

The first effect of introducing a wiggler / undulator as part of a storage ring is that the energy spread of the stored beam is increased due to the quantum nature of the synchrotron radiation emitted by the particles in traversing the device. If the wiggler / undulator is placed in the lattice at a location where there is a finite momentum dispersion there is a consequent increase in beam emittance. This is usually significant only for the higher fields encountered in a wiggler.

Considering the storage ring as a synchrotron radiation source this effect is generally deleterious, decreasing the brightness of the synchrotron radiation beam. For this reason dedicated synchrotron radiation sources often include straight sections in which the momentum dispersion is small or zero. If the storage ring is also used for colliding beam work an increase in emittance may be beneficial since the increased beam size weakens the beam-beam effect and allows higher colliding beam currents and consequently higher luminosity.

If the wiggler / undulator has a net dipole field, then there will be a resulting orbit distortion: therefore, it is conventional to include means of tuning the net dipole field to zero under all conditions of excitation.

As the electrons are perturbed by the wiggler / undulator field they describe a quasi-periodic horizontal motion while traversing the device. Therefore, they cross the edges of the poles and the fringe field region at other than a normal angle. This results in vertical focussing forces on the particles, and the vertical betatron oscillation frequency is correspondingly increased. To get a rough idea of the focussing effects of a wiggler / undulator we consider the simple case of a device with many poles and an essentially sinusoidal field distribution along the axis. We also assume that $(K/\gamma)^2 << 1$ and that the width is sufficient to make the fields seen by the beam essentially two dimensional (i.e., they depend on only two spatial coordinates). Under these circumstances the first order transfer matrix in the bend plane of the wiggler / undulator is the same pure drift matrix that would be present of the wiggler / undulator were absent. In the direction perpendicular to the bend plane, in very good approximation, the diagonal elements of the transfer matrix equal cos ϕ and the off-diagonal elements equal sin ϕ / k_o and $-K_o$ sin ϕ. In these expressions $k_o = B_o/(\sqrt{2}\, B\rho)$, $\phi = L k_o$, B_o and L are the peak field and length of the wiggler / undulator and $B\rho$ is the magnetic rigidity of the electrons.

The magnitude of the tune shift will, of course, vary widely, depending on the energy of the storage ring, the field in the wiggler / undulator and the number of poles. To take two extreme examples for illustration, Helm[28] has calculated that four wigglers each with a field of 1.9 T and a length of 1.5 m in a 4.0 GeV storage ring would produce a vertical tune shift of 0.017, a small perturbation that is . easily compensated or may even be ignored. In a 1 GeV , storage ring, the installation of a single 2 m long 7.2 T (superconducting) wiggler would induce a vertical tune shift of 0.26. This shift in vertical betatron tune would almost certainly have to be compensated by adjustment of the other elements in the lattice or by local compensating elements. In a storage ring with several wigglers each capable of independent adjustment, this tune shift compensation may become complicated.

In cases where the change in local focussing fields caused by the wiggler is compensated by ensemble changes in the lattice, there may be remaining distortions in the betatron function. The wiggler will often be located at other than a symmetry point and it will not easily be possible to re-adjust the lattice to compensate for asymmetry. This lack of symmetry may serve to excite unwanted resonances.

Other effects may arise from unwanted multipole field components; in devices having many poles and short period the measurement of these fields may be problematic.

In the case of the undulator installed in the SPEAR storage ring, a resonance stop band has been observed when:

$$3v_x + v_y = 21 \pm 0.005 \quad (18)$$

(where ν_x and ν_y are the horizontal and vertical betatron wave numbers respectively). Although the presence of this resonance stop band has presented no serious operating problem (the resonance is narrow and easy to avoid), a storage ring with a large number of different devices, each of which presents some prejudice to the operating region, may present difficult operating problems. P. Morton[29] has shown this response to be consistent with the presence of a rotated octupole field component which at 2.5 GeV would have a strength of:

$$\int B \bullet ds = 6x10^{-3} kg \cdot m \text{ at 1 cm} \quad (19)$$

This is a very small field component which also violates mid-plane symmetry. A careful search for this field component has not yet been made. New undulator designs (e.g., the hybrid described elsewhere in this paper) incorporate features for individual tuning of each pole, to preserve mid-plane symmetry.

This should eliminate this resonance. In connection with the proposed installation of a 27 period, high field wiggler at SPEAR,[7] a study of beam stability is being made using a multi-turn ray tracing calculations.[30]

The relative ease with which wigglers and undulators have been incorporated into storage rings is at least in part due to the fact that most of them have had rather large gaps (≥ 3 cm) making it possible to locate them outside the vacuum chamber. With the increasing interest in multipole wigglers and undulators with short periods and high magnetic fields it is clear that smaller magnet gaps offer great advantage.[8] This can be accomplished with a variable aperture device which open to provide the larger aperture required for injection and then closes down to the smaller gaps that are sufficient for the stored beam.

The aperture required for a stored beam depends primarily on the beam size. In an ideal machine (i.e.;

perfect vacuum, no non-linearities, no misalignments, etc.) the transverse beam shape would be gaussian and an aperture of about ± 7σ would be required in a 2-3 GeV ring to maintain a lifetime of 10-20 hours. σ is the standard deviation of an assumed Gaussian and is given by $\sigma = (\varepsilon\beta)^{1/2}$ where ε is the emittance and β is the betaron amplitude function which varies around the ring. The aperture must be at least this large because quantum fluctuations in the synchrotron radiation emission process populate the tails of the beam. The equilibrium beam size is determined by the balance between this quantum excitation and the overall damping provided by synchrotron radiation emission.

For example, in SPEAR, the vertical emittance is $\varepsilon_y = 5 \times 10^{-9}$ m • rad at 3 GeV and the betatron amplitude function at the location of insertion devices is 4.5 meters. Thus $\sigma_y = 0.15$ mm and a full vertical aperture of only about 2 mm would suffice if the vertical beam had a gaussian shape. In practice it is found that the tails contain more particles and a full aperture of 6 mm is necessary .

In addition to the above, it is also necessary to consider the interaction of the stored beam with its environment over the full length of a narrow gap insertion device. As the gap gets smaller more power is deposited by the beam in nearby surfaces. This could result in excessive temperature rise and increased out-gassing.

In addition, the smaller the gap the larger is the effect of image currents on the beam, causing possible possible instabilities. Calculations indicate that these effects do not appear to present serious problems with full aperture of ≥ 5 mm in multi-GeV rings. It is possible that even smaller apertures (~1 mm) may become possible with improvements to existing rings or in newly designed rings. This would open the way for extremely powerful permanent magnet undulators which could reach 10 keV in storage rings with ≤ 2 GeV electron energy.[8]

Because the electron beam cross section is not constant in a straight section, for a magnet of uniform gap, the minimum gap permissible is determined by the largest beam size; i.e. at the end of the magnet ($z = L/2$ where L is the magnet length). The minimum value of the beam size at the end of a magnet can be shown to be

$$\sigma_y\left(\frac{L}{2}\right) = \sqrt{L\varepsilon_y} \quad (20)$$

where ε_y is the vertical beam emittance. At the center of the magnet the beam size is then given by

$$\sigma_{y0} = \sqrt{\frac{L\varepsilon_y}{2}} \quad (21)$$

and the vertical divergence is given by

$$\sigma_{y'} = \sqrt{\frac{2\varepsilon_y}{L}} \quad (22)$$

In the language of accelerator physics we may say that the minimum magnet gap may be used when the value of the vertical amplitude function, β_y, at the center of the magnet is equal to one half the length of the magnet ($\beta_{yo} = L/2$). Smaller values of β_{yo} result in a smaller beam at the center of the magnet but a larger beam at the ends, due to the larger divergence. The beam divergence should also be considered when it is planned to use an undulator in the high on-axis brightness mode. In this case it is desirable to keep the angular divergence of the electron beam small [see equation (14)]. Thus new rings should be built, and existing rings modified, not only to produce small emittance but also to have the optimum value of β at wiggler / undulator locations.

To achieve the smallest possible gap it is desirable to eliminate the thickness of vacuum chamber walls by putting the entire variable gap magnet inside the vacuum system. From the measurements made at SSRL it appears that samarium cobalt material, when properly cleaned and baked, has a sufficiently low outgassing rate that this material could be used for an in-vacuum wiggler / undulator in a high vacuum storage ring.

5. Review of existing and planned wiggler / undulator magnets

A summary of wiggler / undulator magnets in use and in the planning or construction phase in early 1980 was given by Spencer and Winick.[3] Here we will update that material with a brief review of activities at laboratories around the world. We restrict our attention to wiggler / undulator magnets in storage rings and refer the reader interested in work done with synchrotrons to earlier reviews [e.g. reference (3)].

Cornell: An electromagnet wiggler (B_o = 1.8 T, λ_w = 35 cm, 6 poles, K = 59) has been installed and tested[31] in the storage ring CESR, used parasitically for synchrotron radiation by the Cornell High Energy Synchrotron Source (CHESS) Laboratory. At a typical operating energy of 5 GeV the critical energy from this device is about 30 keV compared to 8.5 keV from the ring bending magnets. This magnet is the original SSRL wiggler[26] used in the SPEAR storage ring. The total power requirement is 230 kW.

Frascati: An electromagnet wiggler[32] (B_o = 1.9 T, λ_w = 65.4 cm, 6 poles, K = 116) has been built and is being used in the Adone 1.5 GeV storage ring. The peak critical energy from this device is 2.85 keV compared to 1.5 keV from the ring bending magnets. The magnet gap is 4 cm and the excitation power requirements is 230 kW. A detailed report has been published[27] giving information on the magnet design, magnetic field measurements, effects on the stored beam, beam line and control system design, the properties of the radiation in both wiggler and undulator mode, and the first results of the use of the wiggler for an experiment.

An electromagnet undulator[33] called LELA has been designed and built. It has 20 periods, each 11.6 cm long, a gap of 4 cm and a peak magnetic field of 0.48 T (K = 5.2). The total length is 2.4 m and the total power requirement is 550 kW. The device is intended to be used in a free electron laser experiment.

HASYLAB: Planning is underway[34] to design a 1.2 m long permanent magnet wiggler to be installed in DORIS II late in 1983. Preliminary specifications are B_o = 0.5 T, λ_w = 12 cm, (K = 5.5) magnet gap = 4.2 cm, 10 periods. The new DORIS lattice leaves space for even longer insertions which are planned for later use.

Novosibirsk: A superconducting wiggler[35] (B_o = 3.5 T, λ_w = 9 cm, 20 poles, K = 29) has been constructed and used in the VEPP-3, 2.2 GeV storage ring. The peak critical energy from this device is 11.3 keV compared to 3.8 keV from the ring bending magnets. This device uses a vacuum chamber with a large vertical aperture for the injected beam and a small aperture (8 mm) for the stored beam, thus permitting the attainment of high magnetic field with short period length. The magnet gap is 1.5 cm. After the beam is injected and stored at operating energy it is moved radially from the large vertical aperture part of the vacuum chamber to the small aperture part. The device has been also operated at low electron energy and low magnetic field to produce visible undulator radiation. The interaction of the wiggler with the stored beam has been studied.

A permanent magnet undulator[36] has been built and operated in the VEPP-3 storage ring. The device is part of an optical klystron experiment. It employs $SmCo_5$ material plus steel and consists of 2 separate devices, each containing 3 periods of length 10 cm. The magnet gap can be varied from 1 to 80 cm. It has been operated at a peak magnetic field of about 0.3 T to produce red light spontaneous radiation with the electron energy at 350 MeV. A second version of this device has recently been built and used.[37] It has B_o = 0.7 T and λ_u = 6.5 cm.

On VEPP-4 (5.3 GeV) two wigglers are in use.[37] One is 240 cm long, B_o = 0.8 T and λ_u = 54 cm. The other is 52 cm long and has 3 poles with B_o = 2.3 T.

A helical undulator has been installed on VEPP-2M (0.7 GeV) for high energy physics use.[37] The parameters are B_o = 0.21 T, λ_u = 2.5 em (K = 0.47), 10 periods. The aperture for the beam is only 1.5 cm. Plans are in progress[37] for additional superconducting wigglers for VEPP-2M and VEPP-4.

NSLS: Work on wiggler / undulator magnets at NSLS is reviewed by Hsich *et al.*[38] in these proceedings. For the 2.5 GeV ring two wigglers are in construction. One is 1 superconducting magnet (B_o = 6 T, λ_w = 17.4 cm, 6 poles, K = 97, magnetic gap = 3.2 cm) and the other is a hybrid design permanent magnet (B_o = 1.5 T, λ_w = 13.6 cm, 24 poles, K = 19, magnetic gap = 2.0 cm). Several other devices are planned for the 2.5 GeV ring including a short hybrid wiggler for the 0.1 to 10 keV range and two undulators (one for 0.2-2 keV and the other for 1-6 keV). For the 700 MeV ring a permanent magnet undulator is in construction as part of a free electro- laser experiment (B_o = 0.39 T, λ_u = 6.5 cm, 76 pole, K = 2.4-4, magnetic gap = 2.0 cm) and a UV undulator is planned for the 10-100 e V range.

ORSAY: A 23 period superconducting undulator[16] has been built and used in the ACO 540 MeV storage ring as part of a free electron laser experiment. The period length is 4 cm, the minimum gap is 2.2 cm and the peak field is 0.46 T (K = 1.7). The device has a horizontal field and a vacuum chamber in the shape of an inverted "T," providing a large aperture for injection and a smaller aperture for the stored beam. The entire magnet and vacuum chamber is lowered to permit the beam to enter the magnetic field after injection is complete. Recently the superconducting device was replaced with a permanent magnet device[39] with a period of 7 cm and a maximum value of K of about 2.5. It can be used as an undulator or an optical klystron. Both this and the superconducting device have operated successfully in the storage ring and measurements have been made of spontaneous radiation and gain.

Photon factory: A 3 pole superconducting wiggler[40] with a peak field of about 6 T in the center pole has been built and will be used in the 2.5 GeV storage ring. The peak critical energy from this device is about 75 keV compared to about 4.2 keV from the ring bending magnets. A unique feature of this wiggler is the horizontal magnetic field, producing a vertical deflection of the electron beam and vertically polarized synchrotron radiation

A 10 period permanent magnet undulator[41] has been built and already tested in the 400 MeV SOR ring in Tokyo. The period length is 4 cm, the minimum gap is 2.7 cm and the peak field is 0.125 T (K_{max} = 0.47). Measurements have been made of intensity, brightness and polarization of the radiation. These compare well with theoretical expectations.

SRS: A 3-pole superconducting wiggler[42] with a peak field of about 5 T in the center pole has been installed and operated in the 2.0 GeV ring. The peak critical energy from this device is about 13 keV compared to 3.2 keV from the ring bending magnets.

A 10 period permanent magnet undulator is proposed.[43] The period length is 10 cm, the gap is 4.2 cm, and the peak field is 0.33 T (K = 3.1). The main purpose of the device is to provide the highest intensity in the 100-200 Å region.

SSRL: Now in operation are two electromagnet wigglers.[44] (B_o = 1.8 T, λ_w = 45 cm, 8 poles, K = 76) which were installed into the SPEAR ring in the summer of 1980. Each feeds an X-ray beam line capable of serving 3 experimental stations. The critical energy is 10.8 keV at 3.0 GeV compared to 4.7 keV from the ring bending magnets. The power requirement is about 200 kW and the gap is 3 cm. An earlier 6 pole wiggler[26] was put into operation in late 1978 and is now in use at Cornell.

In construction is a permanent magnet wiggler[7] (B_o = 1.3 T, λ_w = 7 cm, 54 poles, K = 8.5). The device is situated outside a variable gap vacuum chamber which opens to provide the full aperture required for injection and then closes to 1 cm which is adequate for the stored beam.

A 30 period permanent magnet undulator[17] has been built, characterized and used for an experiment. The period length is 6.1 cm, the minimum gap is 2.9 cm and the peak field is 0.24 T (K = 1.4). Detailed information on the performance of this device is given in a separate report[45] in these Proceedings.

The above described undulator plus two new permanent magnet undulators in design will be used for a soft x-ray beam line[5] (10 ≤ $h\nu$ ≤ 1000 eV) at SSRL. These devices will be situated outside fixed vacuum chambers. The parameters of the new undulators have not yet been fixed. An in-vacuum, variable gap, permanent magnet X-ray undulator is planned.[6] The period length is 1.5 cm, the minimum gap is about 7.5 mm and the peak field is 0.35 T (K = 0.5). This device will reach photon energies of about 7 keV in the fundamental peak with SPEAR operating at 3.5 GeV.

6. Beam Line Design Considerations

Beam lines based on wiggler / undulator magnets differ in their design from bending magnet beam lines primarily because of the different spatial distribution of the radiation. A bending magnet produces radiation with a constant spectrum and power density over the full horizontal opening angle of the beam line. The horizontal divergence angle of the beam is usually large enough (10-50 mrad) so that several simultaneously operating experimental stations can be set up in different parts of the beam, even without the use of deflecting mirrors.

Wiggler installations with $K/\Upsilon \geq 10$ mrad offer similar characteristics; i.e., they produce a fan of radiation that is wide enough to serve several simultaneous users spatially separated along the fan. For example, the presently operating SSRL wigglers[44] (B_o = 1.8 T, λ_w = 45 cm, 8 poles, K = 76) produce beams with a full opening angle of about 25 mrad at 3 GeV. The NSLS wiggler[38] now in construction (B_o = 6 T, λ_w = 17.4 cm, 6 poles, K = 97.5) will produce a beam with a full opening angle of about 40 mrad at 2.5 GeV. A similar wiggler[46] planned for the Hefei Synchrotron Radiation Laboratory (HESYRL)[47] in the People's Republic of China would produce an even wider fan (about 100 mrad) because it would be used in an 800 MeV ring.

However, the spectral characteristics of the radiation produced by a wiggler are often not uniform across the horizontal opening angle. This is shown in Figures 5 and 6 for the presently operating SSRL wiggler. The beam line for this wiggler is designed to accommodate 3 experimental stations. The center station can be used with a focussing mirror which accepts ±2.5 mrad about the beam center line. Side stations accept 1 mrad at an angle of 5.25 mrad from the beam center line. The radiation between the center line and the side stations (2.5 mrad on each side) is not used in order to allow for adequate separation of the stations.

Depending on the wiggler field and electron energy the spectrum of radiation delivered to the side stations can have considerably lower critical energy than the center station as shown in the figures. This can lead to operational problems when, for example, the center station, user wants to lower the wiggler field to reduce harmonics. When the wiggler field is near its maximum, all stations receive hard radiation and, of course, all benefit from the enhancement in brightness due to the eight poles.

Depending on the wiggler field and period and also the transverse size of the electron beam in the plane of deflection "single pole" effects can appear in multipole wigglers (i.e.; fluctuations in intensity as a function of angle of observation). For example, if the amplitude of oscillation in the wiggler is greater than the beam dimension in that transverse direction there will be two well separated maxima in intensity in the forward direction and prongs at larger angles due to contribution from single poles or groups of poles. These effects have been analyzed for several wigglers.[48]

Increasing the number of poles to increase the on-axis brightness can result in a fan that becomes too narrow to permit simultaneous operation of more than one station unless deflecting elements are used.

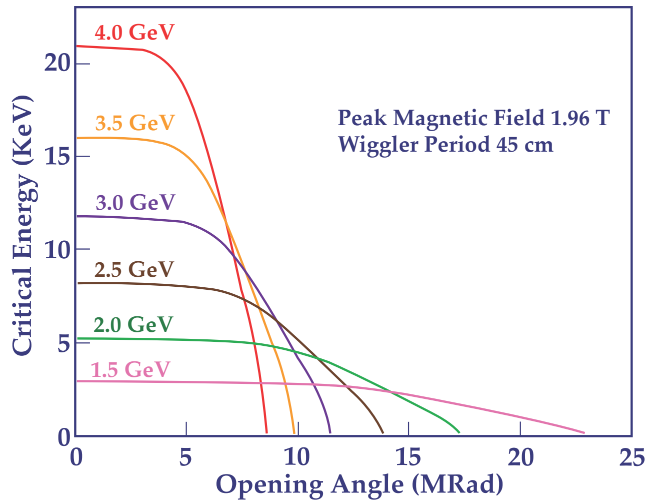

Figure 5 — Critical energy versus opening angle for SSRL Wiggler II. Based on fields calculated using the POISSON program.

This is exemplified in the design[7] of a beam line at SSRL based on a 54 pole, K = 10, wiggler which produces a narrow fan of radiation (2-3 mrad). The center station receives hard radiation while the side stations have a spectrum with a high energy cutoff determined by the angle of incidence on mirrors. Because of the wavelength dependence of the vertical opening angle of the emitted radiation, mirrors inserted vertically may be used to optimize the utilization of long wavelength radiation in the side stations with minimal reduction of short wavelength radiation in the center station.

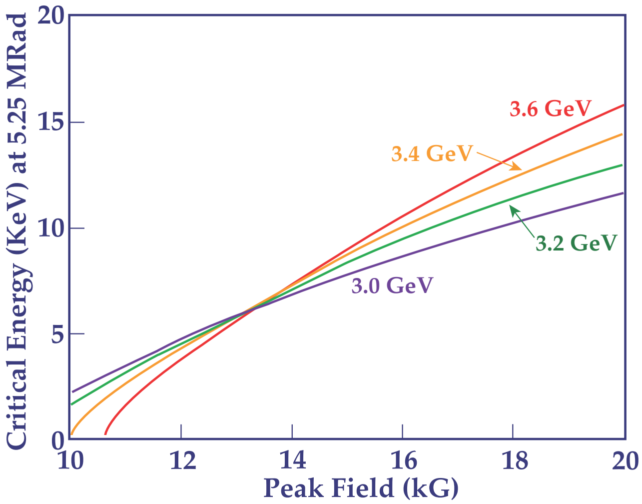

Figure 6 — Critical energy at the side station (5.25 mrad) versus peak magnetic field for SSRL Wiggler II. Based on fields calculated using the program POISSON.

For an undulator ($K < 1$) the beam is even narrower.

This, plus the variation of the wavelength of the quasi-monochromatic peaks with viewing angle, makes it very difficult for simultaneous users to share a single beam line. A design for undulator beam line with deflecting mirrors and three experimental stations is described by Bachrach, *et al.*[5] Only one station receives beam at one time.

Power density considerations are very important in the design of beam lines based on wiggler / undulator magnets, particularly in multi-GeV storage rings. The total power radiated by a wiggler / undulator magnet is given by

$$P = \frac{2}{3}\alpha hc\left\langle \rho^{-2}\right\rangle \gamma^{4}\frac{LI}{e} \quad (23)$$

where I is the stored current, ρ is the instantaneous bending radius and the average is taken over the length L. In practical units

$$P[W] = 1.27E^{2}\left[GeV^{2}\right]\left\langle B^{2}[T]\right\rangle L[mm]I[A] \quad (24)$$

For an N-pole device the peak power along the beam axis (when the amplitude of oscillation is less than the beam size) is given by

$$P\left[\frac{W}{mrad}\right] = 4.33B_{\max}[T]E^{3}\left[GeV^{3}\right]I[A]N \quad (25)$$

where the power is integrated over the full range of vertical opening angles.

Using the above equations one finds that presently operational wigglers in multi-GeV rings produce about 3 kW of total radiated power and 200-300 W/mrad on axis (compared to about 10 W/mrad from bending magnets). Wigglers in construction[7] will produce about 1 kW/mrad on axis resulting in power densities of about 5 kW/cm² on the first mask. New approaches to design of absorbing surfaces must be used to permit operation at these high power levels. Special materials, sloping surfaces, fans in cooling paths and wire mesh absorbers are examples.

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