Specialty Magnets

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Note: The word "iron" is used throughout this paper as a short way to identify soft ferromagnetic materials with large permeability and a large saturation induction. Often, materials such as Vanadium Permendur are used instead.

1. Introduction

The content of this paper is probably not exactly what most people would expect. There are a number of reasons for this. Some of the magnets that one would normally include under the title "specialty magnets" (septum magnets. kickers, …) are covered elsewhere, while I do not have the experience to cover some others that one might expect to find here (lithium lenses, current sheet horns, …). In addition, the editor of this volume has given me great latitude in the choice of subjects that I do, and do not cover. Since permanent magnet (PM) systems are still not used quite as frequently as one might think, I will restrict myself to the magnet systems that use permanent magnet materials (PMM) and can be used either as essential components of accelerators or storage rings, or can be used to do special tasks in such systems. Insertion devices for the production of synchrotron radiation with storage rings of free electron lasers will not be covered because recently they have been the subject of a conference and are described in detail in the proceedings of that conference.[1]

While I occasionally mention papers that describe magnets that have been, or will be built, this paper is not intended to give that kind of review. Instead, I will try to explain a number of basic conceptual designs, without going into specific details or trying to describe all possible variations of the basic designs.

2. Generic Advantages of Permanent Magnet Systems

Figure 1 lists the main generic advantages of PM systems. Of these advantages, I consider the first listed advantage the most exciting: If one scales all linear dimensions of an electromagnet and wants to keep the field strength fixed everywhere, the current density in the coil has to scale inversely proportional to the linear dimensions, leading to insurmountable cooling problems when the linear dimensions become smaller than a value that is specific for each geometry.

Advantages of PM Systems

- Strongest fields when small
- Compact
- Immersible in other fields
- Analytical material
- No power supplies and cooling result in better reliability and greater convenience
- No power bill

Figure 1 — Generic Advantages of PM Systems

If one is forced to reduce the current density in an iron (see footnote at end of paper) free magnet, the field strength will go down as a consequence. In an electromagnet that uses iron in addition to coils, reduction of the current density can be made up by increasing the coil size. However, that leads invariably to increased saturation of the iron. It is this combination of limitations of the current density of the coil and the saturation induction of the iron that leads to a reduction of the fields that can be produced in such a magnet when the linear dimensions need to be small. Since the fields produced by a PM do not change when one scales all dimensions, PM will always outperform electromagnets when the linear dimensions are small enough. The size where this occurs is very different for different geometries. While the critical period length for a wiggler or undulator is of the order 30 cm, the critical aperture for a quadrupole is of the order 1 cm. These numbers hold for anisotropic PMM like Samarium-Cobalt or Neodymium-Iron-Boron with a remanent field B_r of the order .8-1.2 T and a coercive field $\mu_o H_c \sim 0.9 B_r$, and the use of such a material is assumed throughout this paper. As will be shown in Section 4.3, switching to an "all permanent magnet" system is not the only answer: By using PMM judiciously in an electromagnet, its peak field level can reach (or in some cases, surpass) that of a PM system.

While the fact that a PM for a small working aperture turns out to be much more compact than an electromagnet for the same working aperture does not seem to be quite as revolutionary as the stronger field of the PM, the compactness can be, in some cases. just as important: The upper limit of the size of a drifttube in a drifttube linac is controlled by other considerations and therefore makes a PM quadrupole even more attractive for that application.

In the appendix, the magnetic properties of the preferred PMM are described. One of the consequences of these magnetic properties is the following: If I put an iron free PM system into the (time independent) field produced by any other magnet, the resulting field distribution is given, to a very good approximation, by the linear superposition of the fields produced by the individual magnets, and a specific application of this fact will be described in Section 3.2.

Another aspect of the magnetic properties of the PMM used for all magnets described in this paper is the (relative) ease with which one can design PM systems that use these materials. To put this into perspective, I want to point out why it is much more difficult to design a PM system than it is to design an electromagnet. I restrict this discussion to the following two cases: Electromagnets that use iron to control the field distribution and coils to excite the iron; and iron to control the field distribution, and PMM to excite the iron in a hybrid (i.e. magnetically soft plus hard material) magnet. In both cases, the soft material surface "surrounding" the working volume controls the field distribution and the techniques developed for designing these surfaces for electromagnets are, with only minor modifications, directly applicable to hybrid magnets. Assuming that strong saturation of the iron in the electromagnet can be avoided,

$$\Delta V = \int \overrightarrow{H} \bullet \overrightarrow{ds} \quad (1)$$

from one of the poles to the other (e.g. from one pole of a quadrupole to an adjacent pole) is then given exclusively by the number of Ampere-turns provided by the coils and the power supply. Assuming no saturation in the hybrid magnet also, *Δv is affected by every part of the hybrid structure. including the three-dimensional (3D) fringe fields at the ends of the magnet.* Putting it differently: Any change of the geometry or arrangement of either the soft or the hard material will affect ΔV, and with it the field strength and possibly even the field distribution. Or putting it differently again: All the many parameters that describe the whole hybrid magnet system have to work in concert to put every iron block on exactly the right scalar potential. Even if one assumes a long magnet. and therefore ignores the 3D end effects. the design of such a magnet by using one of the finite element magnet analysis codes is an extremely labor intensive process. Even though this is a question of personal style, I find it much more effective and practical to develop analytical models for such magnets and use them for the design. It is extremely fortunate that modern PMM have magnetic properties that permit not only the construction of very strong small magnets. but also make it fairly easy to develop the analytical formulas that one needs to "construct" the analytical models. It is clear that the development of these analytical models leads to a great deal of understanding about the way hybrid magnets function and gives insight into the way each parameter affects the performance of the magnet.

The last points on Figure 1 do not need detailed explanations, except for the following two observations:

1) It is generally underestimated how expensive it is to provide all the components that are needed for cooling (plumbing, low conductivity water. pumps, cooling tower, etc.).

2) For systems that have to go into space, the compactness. the lack of need for power supplies and cooling, translates into enormous reductions of mass.

3. Iron Free Multipoles

3.1 Segmented Dipoles

Figure 2 shows a cross section of a segmented dipole. The dipole field inside a very long such magnet is given by

$$B_0 = B_r \eta \ln\left(\frac{r_2}{r_1}\right) \quad (3.1)$$

with r_1, r_2 indicating the inside and outside radius of the magnet, and B_r representing the remanent field of the PMM (typical values .8-1.2 T).

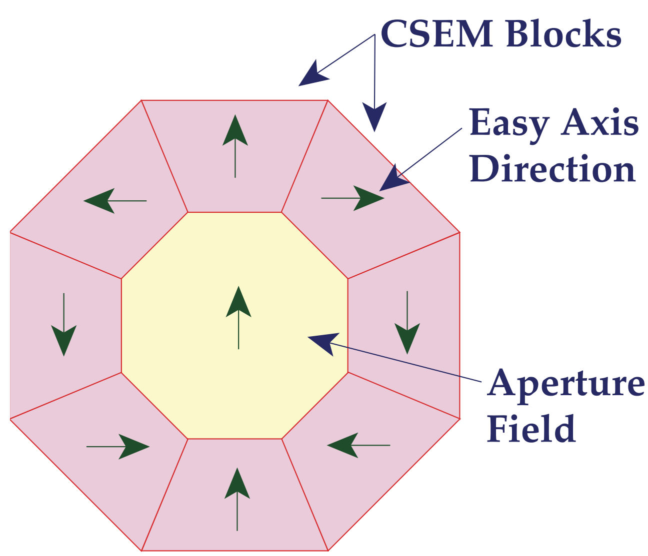

Figure 2 — Segmented PM Dipole

For the derivation of this formula[2] it has been assumed that the differential permeability of the material equals one. If M is the number of blocks in the magnet, η is given by

$$\eta = \frac{\sin\left(\frac{2\pi}{M}\right)}{\left(\frac{2\pi}{M}\right)} \quad (3.2)$$

and $\eta = 0.9$ for $M = 8$

Many iron free PM systems are fairly easy to design. One of the methods that often works is the following: One asks for each infinitesimally small piece of PMM: What is the best orientation of the easy axis so that the piece contributes optimally to the desired field distribution? In particular. if one asks that question for a two-dimensional (2D) multipole (i.e. a magnet whose fields can be derived from a scalar potential

$$V(r,\theta) = a \cdot r^N \cdot \sin(N\phi) \quad (3.2b)$$

one obtains the very simple answer (see Reference 2): The angle α between the easy axis and the x axis at location r, ϕ should be

$$\alpha(r,\phi) = (N+1)\cdot\phi + \frac{\pi}{2} \quad (3.2c)$$

Unfortunately. it is close to impossible to manufacture materials according to this prescription. It is therefore practical to segment the magnet into a number of blocks, each having the same orientation of the easy axis throughout that one should have, ideally, on the center line of each block. This leads to the magnets described in the rest of this section. Other iron free magnets can be designed in the same or similar manner. and some of these have been described in Reference 3.

The maximum achievable field level in the dipole is given by $\mu_o H_k$, with H_k indicating the field where the $B(H)$ curve starts to deviate significantly from linearity (See appendix). The field on axis of a semi-infinite dipole. shown schematically in Figure 3, is given by

$$B(z) = 0.25 B_0 \left[2\ln\left(z + \sqrt{r^2+z^2}\right) + \frac{z}{\sqrt{r^2+z^2}} \right]_{r_1}^{r_2} \quad (3.3)$$

with B_o indicating the field level deep inside the magnet, Equation (3.1), and $B(z)$ given by the difference of the right hand side of Equation (3.3) taken at $r = r_2$ and r_1.

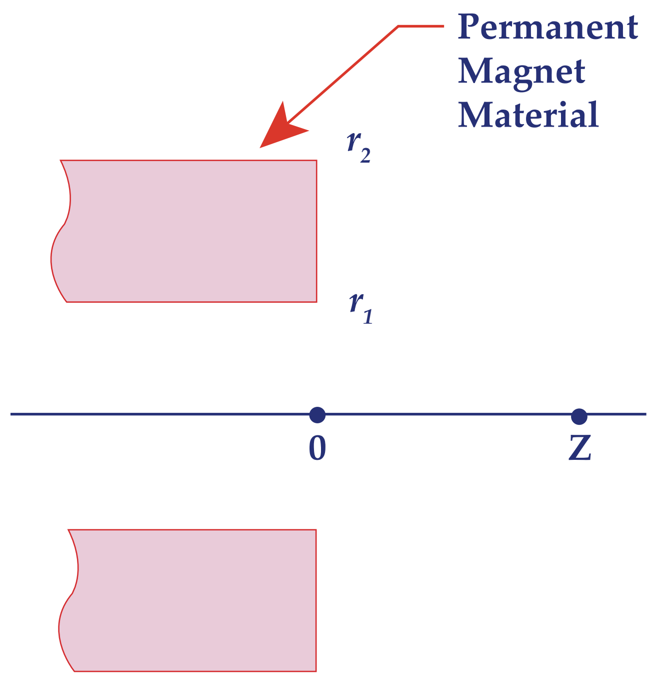

Figure 3 — Semi-Infinite Dipole

It follows from Equation (3.3) that the effective field boundary of the semi-infinite dipole magnet coincides with its physical boundary, giving for a finite length magnet an integrated field of B_o times the physical length.

For the manufacture of this magnet, everything that is stated in Section 3.2 about the more critical quadrupole applies.

The obvious application of this magnet is the "normal" bending of particle trajectories. A not so obvious special application is the use of this dipole as a septum magnet: The fields outside the dipole shown in Figure 2 can be shown to be very small (see Reference 2). Computer runs show that a very thin iron shield reduces the fields in the outside region to a negligible level, making this dipole a very attractive candidate for use as a septum magnet.

Assuming, for simplicity. that the radial thickness r_2-r_1 = D of the magnet is reasonably small compared to r_1 setting $n_\perp = 1$, and using W for the largest possible beam width $2r_1$, the performance limit of this septum magnet is given by

$$B = D \cdot \frac{2B_r}{W} \quad (3.4)$$

For a conventional septum magnet, the performance is given by

$$B = D \cdot \mu_0 j \quad (3.5)$$

where j represents the upper limit for the current density that can be carried by the septum. Comparison of the last two equations shows that the performance limits are different, giving the PM septum the edge unless W is unusually large.

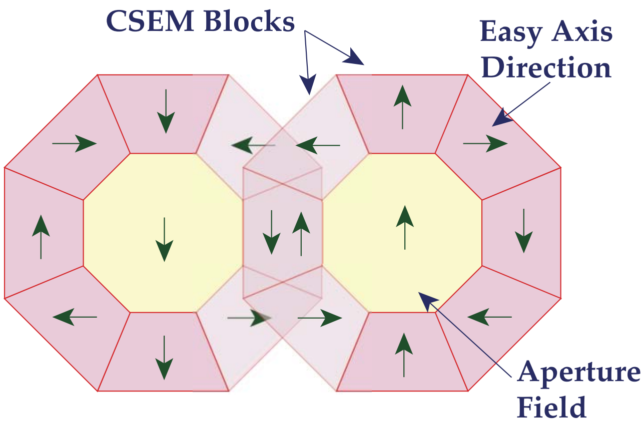

Figure 4 — Double Dipole Septum Magnet

In order to gain a factor two, one could even imagine using a magnet as shown schematically in Figure 4. In the center, two blocks of opposite magnetization direction are shown in the same place. Because of the magnetic properties of the material, one can simply leave out both of these blocks without changing the fields significantly. This leads to what one might call an immaterial septum. i.e. a vacuum region where the fields are so distorted that the beam should stay outside of it. Preliminary computer runs show that this concept may be usable if the demands for field quality in the region seen by the beam are not too stringent. If one wants, as one usually does, no field in one of the two useful regions, one can immerse the whole magnet shown in Figure 4 in a homogeneous dipole field. Another method to gain a factor two will be described in Section 3.2.

3.2 Iron Free Quadrupoles

Figure 5 shows the next member of the multipole family. the segmented quadrupole.

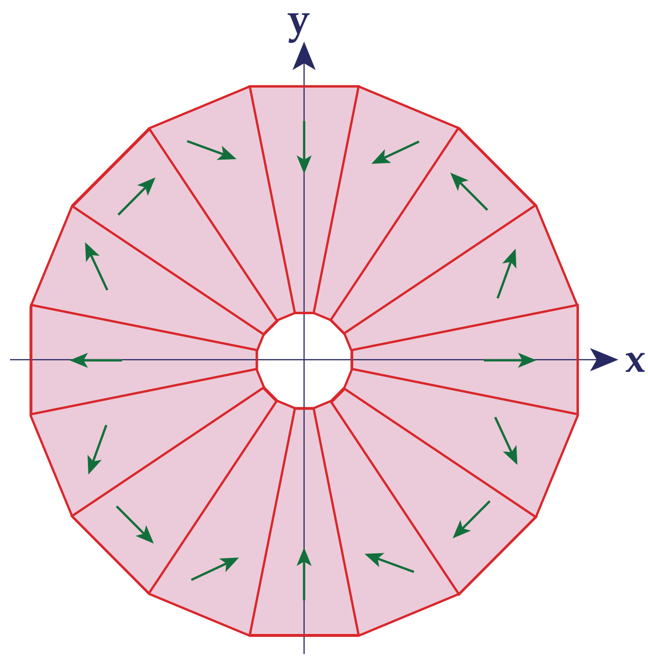

Figure 5 — Segmented PM Quadrupole

The field produced at the aperture radius of this general type of iron free multipole of order N is given by (see Reference 2)

$$B = \frac{N}{N-1} \cdot B_r \cdot \left[1 - \left(\frac{r_1}{r_2} \right)^{N-1} \right] \cdot \eta; \text{ with} \quad (3.6)$$

$$\eta = \left[\cos\left(\frac{\pi}{M} \right) \right]^N \cdot \frac{\sin\left(\frac{N\pi}{M} \right)}{\left(\frac{N\pi}{M} \right)} \quad (3.7)$$

For a 16 block quadrupole (M=16, N=2), η in Equation (3.6) becomes 0.94. The gradient on axis of a semi-infinite quadrupole[3] is given in Reference 3 and the effective gradient boundary of a semi-infinite quadrupole coincides with its physical boundary. The first allowed harmonic in the segmented quadrupole is M+2. By introducing a small gap between the magnet blocks, the first allowed harmonic number can be increased to 2M+2. However, due to manufacturing errors, one usually finds all harmonics. If the field quality has to be very good, it is therefore advisable to mount the blocks in such a way that their position in the assembly can be modified by small amounts. By measuring the harmonics and then making the mechanical adjustments (their magnitude can be calculated using the information given in Reference 4),[4] the field errors can be corrected.

The most obvious application of this type of quadrupole is in a (fixed particle) drifttube linac and in a beam transport line. Consulting Equation (3.6), one sees that with a quadrupole with a ratio of outside to inside diameter of 4, one can reach fields of the order of 1.5 to 1.7 T at the aperture radius, no matter how small the aperture radius is. These quadrupoles could also be used advantageously in medical accelerators and have been used in compact particle spectrometers. Another special application of this quadrupole is its use inside a solenoid of a particle physics detector[5] (see Reference 5): By placing such quadrupoles close to the interaction region of a colliding beam facility, one can increase the luminosity without substantially altering the detector fields.

If one needs a septum magnet. and it is permitted that one has first order focusing in the bend plane, one can use a quadrupole in a manner analogous to the description of the use of a single segmented dipole as a septum magnet. If one considers as the critical quantity the magnetic field just inside the septum. the performance equation for the quadrupole is the same as Equation (3.4), except the right-hand side of that equation has to be multiplied by a factor two. making the quadrupole a very attractive choice under some circumstances.

3.3 Iron Free Sextupoles and Higher Order Multipoles

Figure 6 shows schematically a cross section of a segmented sextupole. Sextupoles (and higher order multipoles) with this kind of peculiar looking arrangement of magnetization directions are very useful in ECR ion sources.

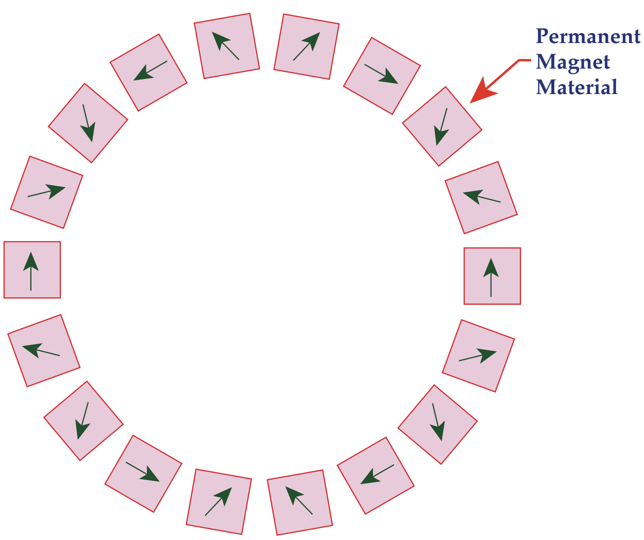

Figure 6 — Segmented PM Sextupole

The multipole magnet serves. in this application. two purposes. It provides the fields for the electron cyclotron resonance, and it also contains the plasma. In a magnet as shown in Figure 6, the region of the vacuum vessel that is heated by the radially escaping particles is easily cooled. Multipoles of this design, being quite strong and compact, are also developed for use as elements in electron storage rings.[6]

4. Hybrid Magnets

While iron free PM have some properties that make them irreplaceable for some applications, they do have some disadvantages that forced the development of hybrid magnets for those applications where the unique properties of the iron free magnets are not essential. The two great disadvantages of iron free PM are the following:

1) Errors in magnetization strength or direction of the blocks translate directly into field errors, i.e. the field quality depends very strongly on material properties.

2) Even though some schemes have been proposed to make the strength of iron free PM systems adjustable, none are really practical, and to my knowledge none have been implemented in devices that are in use.

Both of these disadvantages can be overcome by using iron in addition to the PMM. If one can keep the permeability of the iron sufficiently large compared to one, it does not matter what the permeability is, i.e. the magnetic properties of the iron do not affect the performance. The design, production and assembly of iron to very tight tolerances have been perfected for electromagnets, and these techniques are directly applicable to hybrid magnets.

While the design of the overall geometry of a hybrid magnet is, as mentioned in Section 2, not entirely trivial. the excitation of the iron by the PMM can be made remarkably insensitive to PMM manufacturing errors: One can show that in practically all cases the excitation of the iron is, for all intents and purposes, controlled by the volume integral over the dipole moment density of the PMM. By measuring the total dipole moments of the individual blocks and assigning them to appropriate locations in the assembly, one can minimize the detrimental effects of material manufacturing errors to a level where it is not a serious concern any more. In many cases. one can use the scalar potential bus (see Section 4.1), eliminating this problem altogether.

In the rest of the paper, I use the quadrupole most of the time to explain the fundamental concepts, because it is the most frequently used member of the multipole family. It is worthwhile to point out that most of these concepts can also be used, with very little or no modification of the basic idea, to design combined function magnets.

4.1 Variable Strength Hybrid Quadrupole

Figure 7 shows schematically a variable strength hybrid quadrupole.[7] The four iron blocks are excited by the PMM that is located in the gaps between them and by the PMM that is attached to the outer circular iron ring.

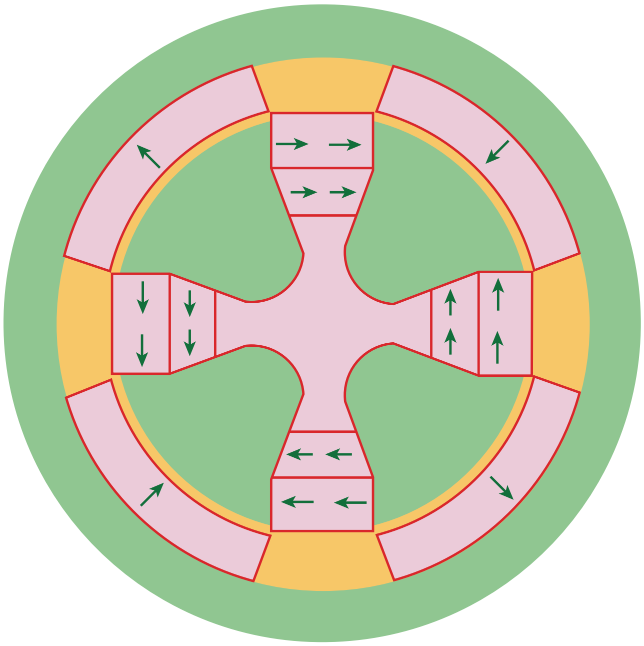

Figure 7 — Variable Strength Hybrid Quadrupole

By rotating this outer ring with the attached PMM one can clearly change the excitation. and with it the gradient. of the quadrupole. If one uses Vanadium Permendur poles, one can reach 1.2-1.4 T at the aperture radius of this magnet. Without imposing variability range as a condition on the design. One usually winds up with a strength range from 100% to 25%. A lower limit of 0% does not penalize other properties of the magnet severely.

In order to avoid field errors. the PMM blocks closest to the aperture should have precise easy axis orientations and only very small gaps are permitted-between the PMM and the poles. Unless one wants the extremely strong aperture fields mentioned above, these blocks can be omitted, leading to a somewhat larger magnet with more PMM in it.

In order to get a magnet with low harmonic content, it is important that there is no scalar potential difference between poles that are supposed to be on identical potentials. i.e. poles that are located diagonally across the center in the case of the quadrupole. While prototype experience shows that this can be achieved by measuring blocks and placing them appropriately, one can make sure by using a scalar potential bus system. By scalar potential bus is meant an iron connection between all poles that are supposed to be on the same scalar potential, thus forcing this condition. Since one cannot allow any bus to be "seen" by the aperture region, it will be installed most of the time at the ends. even though more exotic arrangements are conceivable.

Looking at the whole 3D structure of this hybrid quadrupole, it becomes apparent that in order to get a compact strong quadrupole, one should deposit PMM at the ends and should use field clamps as well.

While the variable strength hybrid quadrupole is less compact and not quite as strong as the iron free PM quadrupole, it is till quite suitable for use in drifttubes and is clearly the best choice if the aperture is small, the field needs to be strong, and variation of the field over a large range is essential. Other obvious uses of this type of magnet is beam transport, medical accelerators and compact spectrometers. It is also worth noting that this type of PM makes it possible to build storage rings where all DC magnets are PM!

4.2 Laced Electromagnetically Tuned Permanent Magnet Quadrupole

If only a small tuning range is required and the advantage of electrical tuning is important, one can use a tuning coil in a PM quadrupole,[8] as shown in Figure 8. In this magnet, alternatingly PMM, a coil and PMM are placed in the gaps between the iron blocks. hence the name laced magnet.

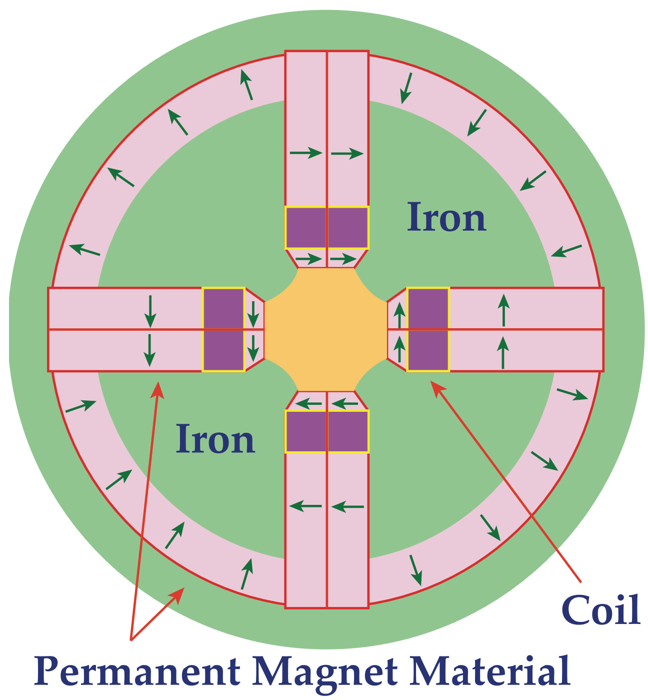

Figure 8 — Laced Electromagnetically Tuned PM Quadrupole

The outer PMM is placed as in the magnet shown in Figure 7 to show that one could build a magnet that has coarse mechanical tuning and fine (fast!) electrical tuning. If compactness is not important and mechanical tuning is not needed, it would probably be more convenient to have an outer iron shell with attached PMM that is square instead of round.

It should be noted that the tuning coil should be placed close to the aperture region, as shown in Figure 8, in order to keep its size down. This is important because the larger the coil has to be. the more PMM will be needed. and the larger the overall size of the magnet will be. For that reason, it would also be advantageous to use a bipolar power supply for tuning, cutting the coil by about a factor two in size.

This type of quadrupole is clearly suitable for the applications listed at the end of Section 4.1, but can be used additionally in situations where rapid variation of the field strength is important.

4.3 The Laced Electromagnetic Quadrupole

Under some circumstances. the limitations of the magnets described in Section 4.1 and 4.2 may be undesirable.

For that reason. an electromagnet that does not suffer the field strength reduction associated with small dimensions would be a desirable device. While undulators and wigglers with these properties have been developed,[9] the underlying concepts were somewhat limited and application and device specific. The method of use of PMM in an electromagnet described here is. in contrast, applicable to *any* electromagnet. To explain the concept, I use again the quadrupole as the most important member of the multipole family. The following explanation is intentionally detailed since the use of PMM in an electromagnet is somewhat unusual and. consequently. very unfamiliar to most people.

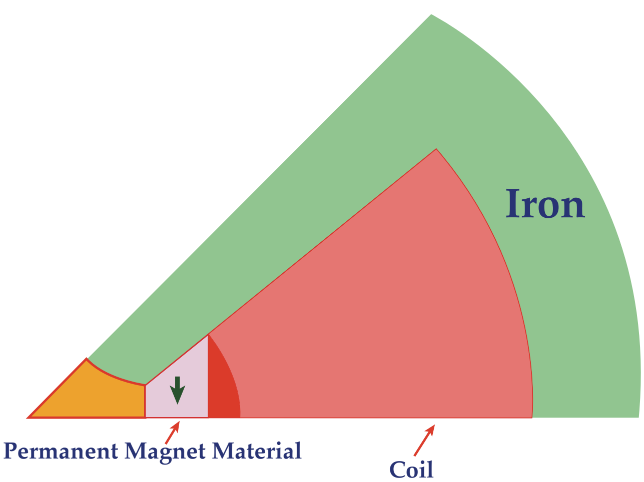

Figure 9 — 45° Slice of a Laced Quadrupole

Figure 9 shows schematically a 45° slice of the cross section of a quadrupole that uses both a coil and PM. In a conventional small aperture electromagnetic quadrupole, a lot of flux enters the pole in the region just beyond the shaped pole contour. For fixed current density in the coils and fixed field at the magnet aperture, the resulting saturation of the iron increases as the aperture decreases. By placing PMM into this region, as shown in Figure 9 and in the field line pattern plot in Figure 10, this flux is dramatically reduced, making it possible to reach the same aperture fields quoted for the hybrid quadrupole.

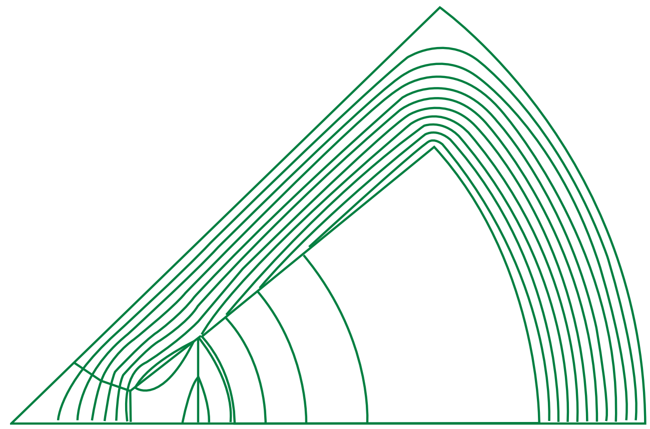

Figure 10 — Field Lines in a Laced Quadrupole

This basic concept can, of course, be modified in many beneficial ways: one can tilt the PMM block, one can use a direction of the easy axis that is not parallel to the edge of the block, one can put PMM even closer to the aperture than shown in Figure 9, one can put PM at the ends of the quadrupole, etc. In extreme cases (usually not in quadrupoles, but definitely in magnets like undulators / wigglers) it is even necessary to have alternatingly several blocks of PMM and coils.

That this type of magnet behaves like an electromagnet is clear if one takes

$$\int \vec{H} \cdot \vec{ds} \quad (4.3a)$$

along the boundary of the magnet shown in Figure 9. Application of Ampere's law, and using I for the total Ampere turns in the part of the magnet shown in Figure 9, yields

$$\left(\int H_r \, dr \right)_{ap} = I - \left(\int \vec{H} \cdot \vec{ds} \right)_{iron} \quad (4.3b)$$

This equation shows clearly that the presence of the PMM is felt only indirectly: The only term that is affected by the presence of the PMM is the last term, representing the loss of excitation due to saturation of the iron. The indirectness of the effect of the PMM does not mean that the effect is small: The field strength of a small aperture hybrid drifttube quadrupole can be improved by a factor 1.5, and the gain can be significantly larger for other structures. Since the iron is preloaded with magnetic flux generated by the PMM, the saturation curve of the magnet is extremely asymmetric, allowing much higher fields in the "forward" direction than in the reverse direction. While this does generally not represent a problem for drifttube quadrupoles, this effect can be quite extreme in other systems. In strong laced undulators, it is not even possible to turn the device off without encountering severe saturation effects.

4.4 Multiaperture Quadrupole System

In order to reduce unwanted space charge effects when one has to accelerate large beam currents, multiple aperture accelerators have been considered for quite a while. Such an accelerator needs multiple aperture focusing systems. Since the field errors resulting from errors in strength and easy axis direction of PMM are much more difficult to handle in a multi aperture focusing system, I describe here only a hybrid multiple aperture focusing system.

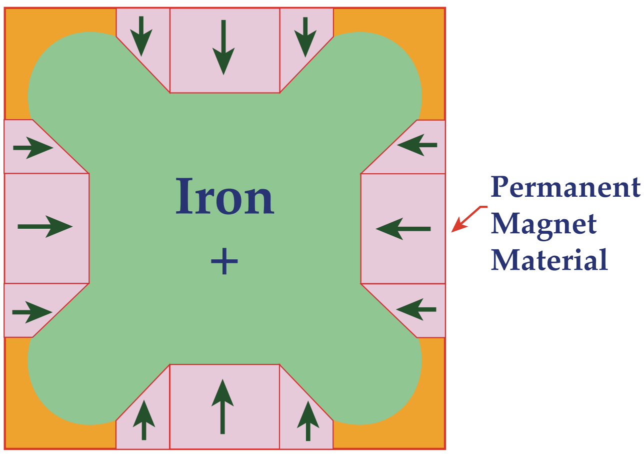

Figure 11 — Basic Module for
Multiple Aperture Quadrupole System

Figure 11 shows the basic element of the whole system, and Figure 12 shows a whole system. In order to assure proper excitation of all poles, it is advisable to use a scalar potential bus system, i.e. two systems of iron connectors at the ends that magnetically connect all the iron blocks that have to be on the same scalar potential. If one has to handle a large number of beams, the unusable partial apertures at the periphery of the system do not increase the system cost and size significantly, and one can build the system as shown in Figure 12, with the outer boundary being the boundary of an iron yoke: If one has to accelerate only 9 beams or less, one can save some expense and size by cutting the system off along the dashed line, placing PMM appropriately along that line and then surrounding the whole system again by an iron frame. As with all the hybrid magnet systems, this one can be designed quite easily with analytical methods.

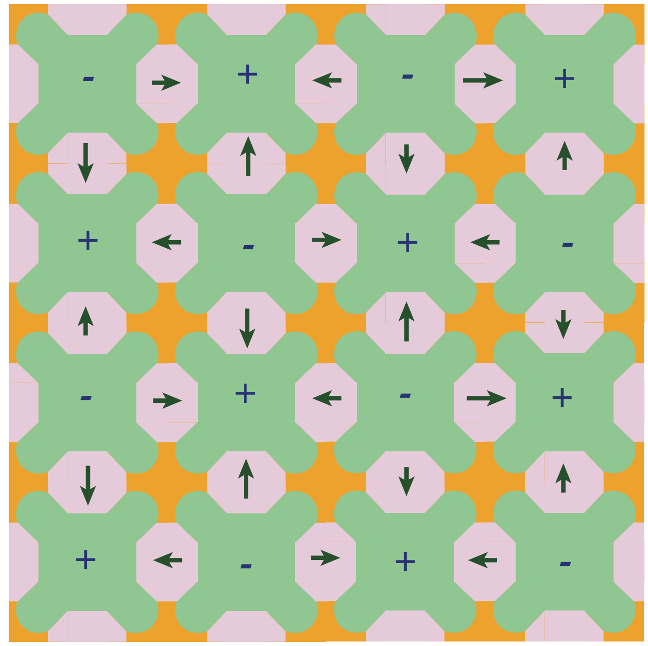

Figure 12 — Multiple Aperture Quadrupole System

4.5 The Box Dipole

Figure 13 shows schematically a cross section through a hybrid dipole that is quite practical and useful for a number of applications, in particular compact spectrometers with deflection angles that are so large that the magnet has to be curved. While changing the field level by a large factor is difficult, fine tuning the field is fairly easy by moving or rotating blocks of PMM.

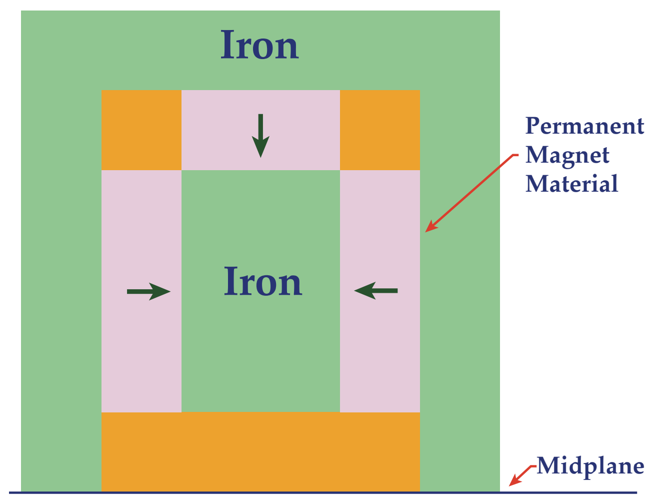

Figure 13 — Box Dipole

4.6 The Hybrid Solenoidal Field Doublet

Solenoids are very effective focusing elements for low momentum beams. If one produces such fields with a PM system, Ampere's law tells us that

$$\int_{-\infty}^{\infty} H_z(z)\,dz = 0 \quad (4.6)$$

Remembering also that the local focusing strength of a solenoidal field is proportional to B_z^2 leads to a symmetric doublet as the most economical and compact system, and a schematic cross section of such a doublet is shown in Figure 14.

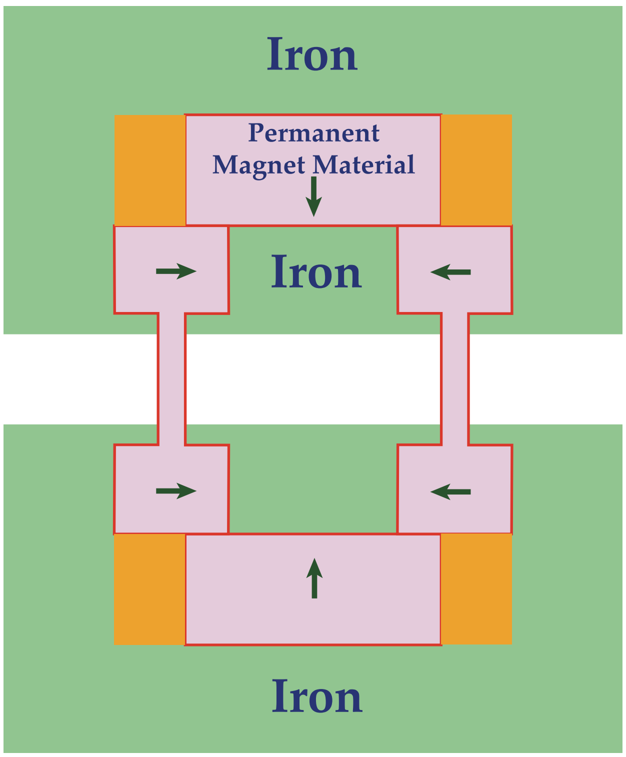

Figure 14 — Hybrid Solenoidal Field Doublet

Preliminary studies show that one can achieve on the axis peak fields of about .8-1T. The system shown in Figure 14 does not incorporate field variability, but it should not be too difficult to incorporate that feature into the basic design.

Appendix

Manufacturing Process and Magnetic Properties of Some Permanent Magnet Materials

To get a rough understanding of the qualitative aspects of the magnetic properties of some permanent magnet materials (PMM), I describe very crudely the powder technology that is often used to make some of the modern materials like Samarium Cobalt or Neodymium-Iron-Boron. This is then followed by a brief description of the magnetic properties of these materials.

After a molten mixture of the properly chosen ingredients is solidified by rapid cooling, a crushing and milling process produces a powder that consists of grains of well controlled size. If done properly, each grain is a ferromagnetic domain with an extremely strong crystalline anisotropy and is always magnetically polarized in the direction parallel to the crystalline axis. Exposing that powder to a strong homogeneous magnet field and giving the grains the opportunity to macroscopically orient themselves in the field (ultrasound, lubricants, and other means are used), one has produced a powder in which the crystalline axes of all grains align. After increasing the pressure in order to increase the density and to obtain a solid (fragile) block of material. one sinters the block and machines and grinds it if necessary. Since the sintering temperature is above the Curie temperature, the blocks are finally exposed once more to a magnet field parallel to the orientation field direction, so that "all" ferromagnetic domains are magnetized in the same direction, the so-called easy axis.

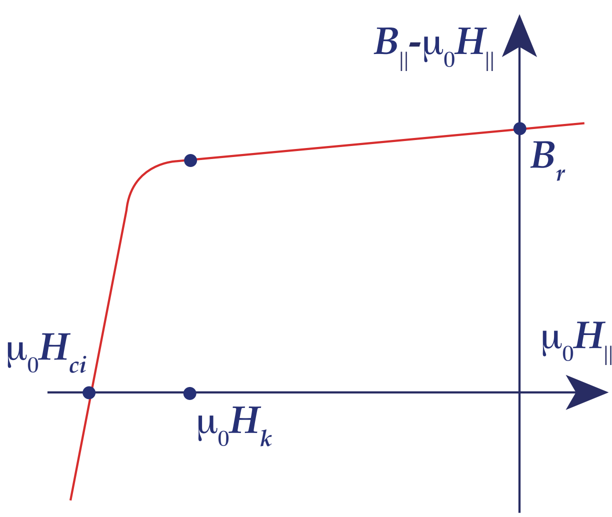

Figure 15 — Magnetization Curve

Figure 15 shows the resulting magnetization curve. Starting with the large value of $H_{\|}$ used during the final magnetization and reducing $H_{\|}$ it is not terribly surprising that the magnetization $B_{\|} - \mu_o H_{\|}$ does not decrease significantly while $H_{\|} > 0$. The remarkable property of these materials is the fact that, because of the extremely strong crystalline anisotropy of the grains that make up the macroscopic block, the magnetization remains essentially constant until a substantial field is applied in the reverse direction. The value of H at the knee in the curve where one starts to lose a significant fraction of the magnetization is called H_k, and the field where the magnetization $B_{\|} - \mu_o H_{\|} = 0$ is called the intrinsic coercive field H_{ci}. As long as one stays to the right of the knee, one can (possibly after an initial small irreversible loss of magnetization) reversibly move back and forth on the magnetization curve, while one suffers an irreversible magnetization loss if one goes beyond H_k. This loss can be recovered only by magnetizing again strongly in the forward direction.

Constructing from the curve shown in Figure 15 the curve $B_{\|}$ vs. $\mu_o H_{\|}$ shown in Figure 16, the field where $B_{\|} = 0$ is called the coercive field H_c. From this description, it is clear why it is not advisable to drive the material beyond the knee H_k: One loses dipole moments. and the material properties become quite non-linear.

For rare earth PMM. typical values for B_r are .8-1.2 T. In the straight r part of the $B_{\|}(H_{\|})$ curve, the differential permeability $\mu_{\|}$ ranges from 1.03 (for "good" Samarium Cobalt) to about 1.15 (some of the Neodymium Iron materials). The magnetization curve $B_{\perp}(H_{\perp})$ in the direction perpendicular to the easy axis goes through the origin, is usually quite straight and has a value for its permeability $\mu_{\perp}$ that is not very different from $\mu_{\|}$. For many design considerations, and all discussions in this paper, I assume $\mu_{\|} = \mu_{\perp} = 1$.

The material property that is very valuable for our applications is H_k: I practically never permit the use of materials with $\mu o H_k < B_r$, since accidentally driving the material beyond the knee obviously damages it magnetically. For some applications, like the strong iron free multipoles discussed in Section 3. one needs substantially larger values of H_k. Some commercially available Samarium Cobalt has B_r ~ .9 T, and $\mu o H_k$ ~ 1.8 T.

It should be pointed out that while the rare earth PMM are used most often for accelerator applications, some of the ferrites have qualitatively the same properties that the rare earth PMM have, but have a much smaller remanent field, namely B_r ~ .3-.35 T. The advantage of these materials is their much lower cost: A study I undertook a few years ago showed that a hybrid dipole magnet with a field of 2.1 T was quite inexpensive if one used ferrite PMM, while a hybrid with Samarium Cobalt was very expensive.

Using the magnetic properties described above in the magnetostatic equations. and assuming that one does not go beyond Hk, one can easily show (2) that the magnetic properties of these materials can be described as follows: The material has a (passive) anisotropic permeability close to one, and either an “imprinted” (active) current density curl

$$\frac{\vec{B}_r}{\mu_0 \mu_{\|}} \text{ (A1)}$$

or an “imprinted” (active) magnetic charge density

$$-div\left(\vec{B}_r\right) \text{ (A2)}$$

In these expressions, $\vec{B}_r$ represents the local magnetization by magnitude (B_r) and direction (of easy axis). Since it is unfortunately very difficult to (intentionally-) make materials that are inhomogeneously magnetized, we deal practically exclusively with a $\vec{B}_r$ that is constant in magnitude and direction inside the material. Consequently. the active properties of such blocks can be represented by either current sheets or charge sheets on the surface, leading to the generic term charge (or current) sheet equivalent material (CSEM).

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