Perturbation Effects in Segmented Rare Earth Cobalt Multipole Magnets

By Klaus Halbach, Lawrence Berkeley Laboratory,
University of California, Berkeley, CA 94720

General formulae are given that allow calculation of harmonics produced by some fundamental perturbations of the components of segmented 2D rare earth cobalt multipole magnets. Also given are relevant specific formulae for the most frequently used shapes of components of segmented multipoles.

1. Introduction

As segmented rare earth cobalt (REC) multipole magnets[1] are used more frequently, the need arises to evaluate the production of undesired harmonics by perturbations and, conversely, to "fix" undesired harmonics by intentionally perturbing segments of the magnet. Considering only two dimensional (2D) fields and perturbations, the relevant formulae are easily derived from reference 2,[2] and this is done here to save the reader the effort of reading a major part of reference 2.

2. Notation and Summary of Some Results of Reference 2

We assume throughout that the magnetic properties of the permanent magnet material can be described with sufficient accuracy by

$$B_{\|} = B_r + \mu_0 H_{\|} \quad (1b)$$

in the direction parallel to the easy axis, and by

$$B_{\perp} = \mu_0 H_{\perp} \quad (1c)$$

in the direction perpendicular to the easy axis. Since these conditions are reasonably well satisfied for REC, we talk about REC, even though some other materials satisfy these conditions fairly well also.

Using complex notation, the 2D fields produced by a uniformly magnetized block of REC that is located outside the coordinate origin can be described by

$$B^*(z_0) = B_x - iB_y = \sum_{n=1}^{\infty} b_n z_0^{n-1} \quad (1)$$

and the b_n can be calculated from

$$b_n = \frac{\mathbf{B}_r}{4\pi i} \int \frac{dz^*}{z^n} \quad (2)$$

In these equations, $z = x + iy$, $z^* = x - iy$, and

$$\mathbf{B}_r = B_r e^{i\beta} \quad (3)$$

with β representing the angle between the easy axis in the REC and the x axis. The integral in equation (2) is to be taken over the boundary delineating the block of REC.

From equation (2) follows that for a block that is physically rotated about the origin by the angle α; and whose easy axis forms the angle (β + γ with the x axis, the b_n has the relationship

$$b_n(\alpha,\gamma) = b_n(0,0)e^{i[\gamma-\alpha(n+1)]} \quad (4)$$

with the original block, which we designate now as the reference block.

If one assembles a segmented multipole magnet (figure 1 shows schematically a segmented quadrupole) from M geometrically identical blocks, with block m ($1 \le m \le M - 1$) obtained from the reference block ($m = 0$) by rotating it by the angle $m 2\pi/M$ about the origin, and its easy axis forming the angle $\beta + m \cdot (N+1)\, 2\pi/M$ with the x axis, then the field produced by this segmented multipole magnet is obtained from equation (1) by multiplying the right side by M and taking the sum only over

$$n = N + \mu \cdot M, u = 0,1,2,3\ldots \quad (5)$$

thus giving a magnet whose field B^* is dominated by the term $M \cdot b_N z_0^{N-1}$.

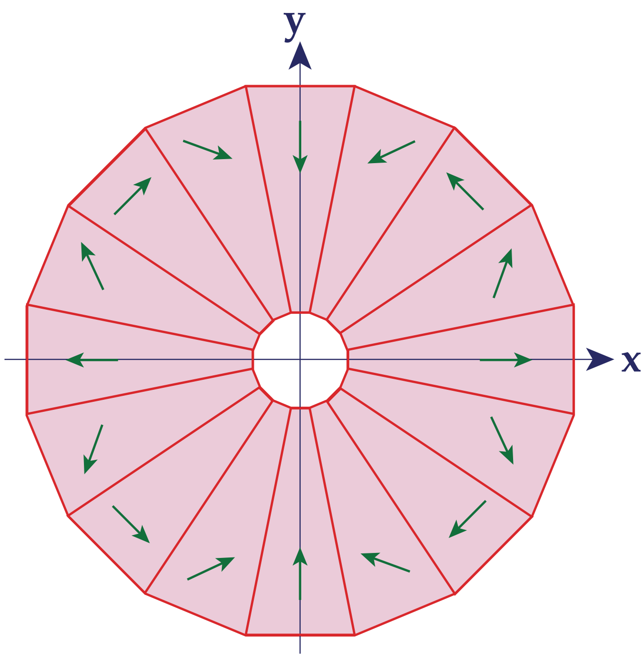

Figure 1 — Segmented quadrupole

3. First Order Field Perturbations Caused by Small Perturbation of the Reference Block

We consider 5 different possible perturbations of the reference block: Perturbation of (1) the magnitude of magnetization by ΔB_r; (2) the direction of the easy axis by $\Delta\beta$; the linear displacement of the reference block by (3) Δx and (4) Δy (describable by displacement Δz); (5) location of the block by rotation about the origin by $\Delta\alpha$, with the easy axis remaining fixed in the coordinate system of the block. In each of these cases, the first order perturbation effect on the harmonics produced by the reference block is easily and directly obtained from equation (2), and one obtains

$$\Delta b_n = \frac{\Delta B_r}{B_r} \cdot b_n \quad (6)$$

$$\Delta b_n = i \cdot \Delta\beta \cdot b_n \quad (7)$$

$$\Delta b_n = -n \cdot \Delta z \cdot b_{n+1} \quad (8)$$

$$\Delta b_n = -i \cdot n \cdot \Delta\alpha \cdot b_n \quad (9)$$

4. Field Perturbations Caused by Blocks Other Then the Reference Block

Identifying the reference block by $m = 0$, block m of a magnet consisting of M segments is rotated about the origin by $\alpha = m \cdot 2\pi/M$ relative to the reference block, and the easy axis is rotated by $\gamma = m(N+1)2\pi/M$ relative to the easy axis of the reference block. Equation (4) therefore leads for all perturbations described by equation's: (6)-(9) to:

$$\left(\Delta b_n\right)_{Block\ m} = \left(\Delta b_n\right)_{\text{Ref. Block}} \cdot e^{-i\delta_m} \quad (10a)$$

$$\delta_m = \frac{m 2\pi (n - N)}{M} \quad (10b)$$

Notice that the displacement Δz of block m is defined in a coordinate system that rotates with the block, i.e. in the coordinate system of the block itself.

5. Consequences of Equation (10) for Intentional Generation Harmonics

Under some circumstances, it may be desirable correct existing (measured) harmonics by intentionally perturbing some blocks in the same manner by differing amounts. This procedure will in general only be satisfactory if one can produce these harmonics with any desired phase angle. Unfortunately, equation (10a) limits this: If M is even, it follows from equation (10b) that δ_m is a multiple of π if $n = N + \mu \cdot M_2$, $\mu = 1, 2, \ldots$, thus precluding in particular the production of an arbitrary phase angle of b_n for $n = N+M/2$.

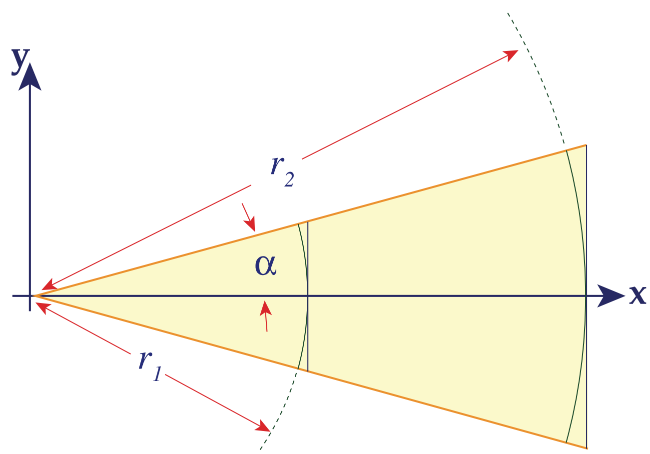

Figure 2 — Reference block of trapezoidal shape

6. b_n Produced by Some Specific Blocks

From equation (24a) of reference 2, b_n for a reference block trapezoidal shape, as shown in figure 2, is given by

$$b_n = \frac{\mathbf{B}_r}{\pi} \cos^n(\alpha) \frac{\sin(n\alpha)}{n-1} \left(\frac{1}{r_1^{n-1}} - \frac{1}{r_2^{n-1}} \right) \quad (11)$$

If the inside and outside contours of the circular arcs, equation (24b) of reference 2 gives

$$b_n = \frac{\mathbf{B}_r}{\pi}\frac{n}{n^2-1}\sin\left[(n+1)\alpha\right]\left(\frac{1}{r_1^{n-1}}-\frac{1}{r_2^{n-1}}\right) \quad (12)$$

In both cases,

$$\frac{1}{n-1}\left(\frac{1}{r_1^{n-1}}-\frac{1}{r_2^{n-1}}\right) \quad (12b)$$

is to be replaced by $\ln(r_2/r_1)$ for $n = 1$.

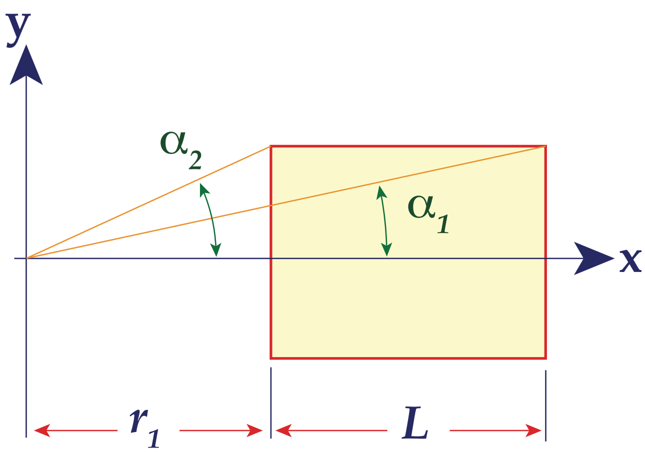

Figure 3 — Reference block of rectangular shape

If a multipole magnet does not have to be very strong, it can be rather cheaply assembled from REC blocks of rectangular cross section. Such a block (see figure 3) produces the harmonics

$$b_n = \frac{\mathbf{B}_r}{\pi}\frac{1}{n-1}\frac{1}{r_1^{n-1}}\left\{\begin{array}{l}\cos^{n-1}(\alpha_2)\sin\left[(n-1)\alpha_2\right] \\ -\left(\dfrac{\cos(\alpha_1)}{1+L/r_1}\right)^{n-1}\sin\left[(n-1)\alpha_1\right]\end{array}\right\} \quad (12c)$$

with

$$\tan(\alpha_1) = \frac{\tan(\alpha_2)}{(1+L/r_1)} \quad (12d)$$

For a block with circular cross section that does not enclose the origin (see figure 4) and that one may want to use just for tuning purposes, equation (2) gives:

$$b_n = \mathbf{B}_r \frac{n}{2}\cdot\frac{r_2^2}{x_1^{n+1}} \quad (13)$$

and equation (4) allows calculations of the effect of such a block when located at an arbitrary location.

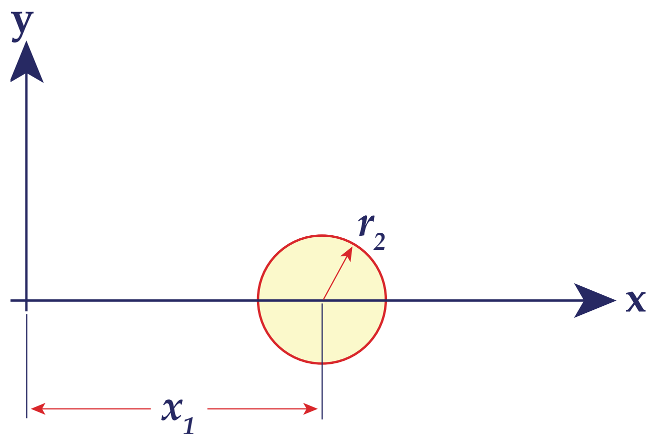

Figure 4 — Reference block of circular cross section

Publishing History

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