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# Permanent Magnets for Production and Use of High Energy Particle Beams

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In the last few years, permanent magnet systems have begun to play a dominant role in the generation of synchrotron radiation and the operation of free electron lasers. Similarly, permanent magnets can lead to significant improvements of accelerators and systems that use them. The general conditions will be discussed under which one can expect benefits from permanent magnets, and a number of specific applications will be described in detail.

## 1. Magnetic Properties of Some Permanent Magnet Materials

All permanent magnets described in this paper use anisotropic material whose magnetic properties are adequately described by

$$B_{\parallel} = B_r + \mu_0 \mu_{\parallel} H_{\parallel}; H_{\parallel} : B_{\parallel} > 0 \quad (1)$$

$$B_{\perp} = \mu_0 \mu_{\perp} H \quad (2)$$

In these equations,  $\parallel$  and  $\perp$  refer to the direction parallel and perpendicular to the preferred direction of the material, the so-called easy axis. Depending on the application, it is necessary that equation (1) holds well into the second quadrant (and for some strong magnets without the use of iron even in the third quadrant) of the B-H coordinates. It is further assumed that  $\mu_{\parallel-1}$  and  $\mu_{\perp-1}$  is smaller than 0.1 ( $\leq 0.05$  for Rare Earth Cobalt). The materials that satisfy these conditions are Rare Earth Cobalt ( $B_r = .8 \text{ T}$ ), Neodymium Iron Boron ( $B_r = 1.2 \text{ T}$ ), and some of the ferrites ( $B_r = .2-.35 \text{ T}$ ). Even though the first two mentioned materials are the most powerful permanent magnets materials, strength is not the only reason for preferring these kinds of materials for the design of magnets for very demanding applications: The above mentioned properties allow, as a very good approximation, the application of linear superposition of the effects of different blocks of permanent magnets material (as long as there is not strongly saturated iron present). The resulting simple theory gives a very good understanding of the properties of systems composed of these materials, and good designs follow rather easily from that good understanding.

It is easy to show that the magnetic field produced by a uniformly magnetized block of permanent magnets material with the above described properties is, in very good approximation, the same as the field produced by either currents or charges on the surface of the block. For that reason, we refer to this class of material to current sheet equivalent material or charge sheet equivalent material (CSEM).

## 2. Generic Advantages of Permanent Magnets

Before describing some specific devices, it is useful to discuss the general circumstances under which the use of permanent magnets is indicated, and what the preferable permanent magnets materials are.

When one scales an electromagnet in all dimensions while keeping the magnetic field at equivalent locations fixed, it is easy to see that the current density in the coils scales inversely proportional to the linear dimensions  $L$  of the magnet. Since superconductors have an upper limit for the current density  $j$  that can be carried and dissipative coils have an upper limit for  $j$  due to the need to remove the dissipated power,  $j$  needs to be reduced below that prescribed by simple scaling when  $L$  reaches a certain small value that depends, of course, on many details of the magnet design. When  $j$  is reduced, the field in a magnet that does not use iron obviously is also reduced, even if the total ampere turns are maintained by increasing the coil size. The same is also true for a magnet using iron, since an increase of the coil size invariably leads to a field reduction due to increased saturation of the iron. Permanent magnets on the other hand, can be scaled to any size without any loss in field strength. For the same reasons, small permanent magnets systems will be much more compact than electromagnets of equal, or even lesser, performance. From this follows that *when it is necessary, magnetically significant, dimension of a magnet is very small, a permanent magnet will always produce higher fields than an electromagnet.* This means that *with permanent magnets one can reach regions of parameter space that are not accessible with any other technology.* The critical size below which the permanent magnets out-performs the electromagnet depends of course

on a great many details of both the desired field strength and configuration as well as the properties of the readily available permanent magnets materials. In the region of the parameter space that is accessible to both technologies, the choice of one technology over the other will be made on the grounds of cost, reliability, or convenience. (main specifics: power supplies, power needed to run the system, equipment associated with cooling) and in this arena permanent magnet systems are often also preferable, but in general less so the larger the smallest magnetically relevant dimension becomes.

### 3. Magnetic Lenses and Related Magnets for High Energy Accelerators and Storage Rings

#### 3.1 Reasons for Use of Magnetic Lenses

When charged particle beams travel very large distances (in some storage rings, beams circulate for hours with velocities close to that of the velocity  $c$  of light), the particles obviously need to experience an average force directed toward some axis that represents the ideal trajectory. Without this focusing force, the particles would very soon hit the vacuum chamber and be lost for the intended use. This force can be produced either by application of a transverse electric field  $E$  or by a transverse magnetic field  $B$  that produces a force equal to that of the equivalent electric field  $E_{eq} = \tau v B$ , with  $\tau v$  representing the velocity of the particle. If  $\tau v > E_m/B_m$  where  $E_m$  and  $B_m$  are maximum values of  $E$  and  $B$  that can be produced with a reasonable effort, the magnetic focusing force is larger than that achievable with electric fields. Assuming  $B_m = 1\text{T}$  and (for electrode distances of the order of 1 cm)  $E_m = 10^5\text{ Vcm}^{-1}$ , the critical value for  $\beta = \tau v/c$  is of the order .03. For this reason, magnetic focusing is used exclusively for focusing of high energy charged particle beams.

Most accelerator magnets are considerably longer than the radius of their useful field aperture. For that as well as some more sophisticated reasons, the properties of most interest are the two dimensional (2D) aspects of the magnetic fields produced by these magnets, and we will discuss only these aspects here.

Nearly all accelerator magnets are multipole magnets whose 2D fields can be derived from a scalar potential of the form

$$V = \text{Constant } r^n \cos(n\phi); \quad n = \text{integer} \quad (3)$$

Among these magnets, those with  $n = 2$  (quadrupoles) are used most often, closely followed by those with  $n = 1$ . Corrective magnets with  $n = 3, 4$  are also used, but in much smaller numbers.

#### 3.2 Magnetic Lens Properties

Examination of the particle motion caused by a quadrupole lens shows that while the force in one transverse direction is focusing, it is defocusing in the transverse direction perpendicular to that direction. By using a properly separated pair of lenses with opposite excitations, one gets a net focusing force for this alternate gradient focusing system. This is clear from Figure 1 when one takes into account that the focusing/defocusing force in each lens is proportional to the distance of the trajectory from the axis.

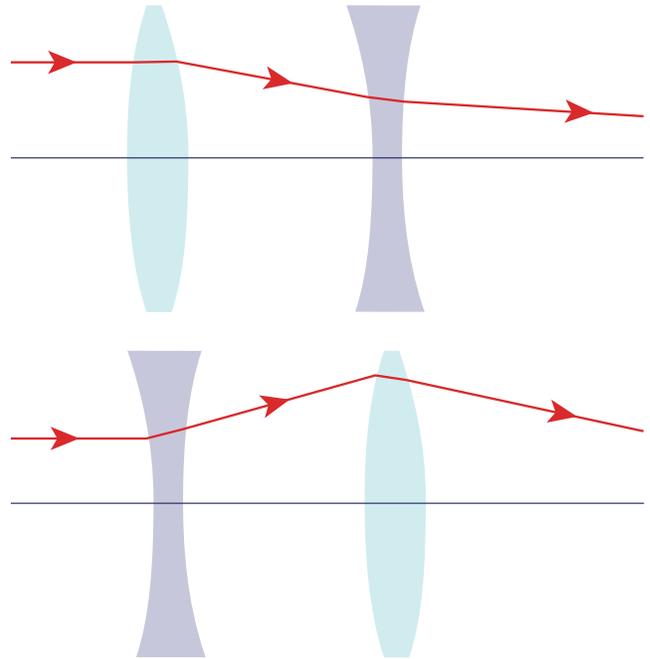


Figure 1 — Net Focusing by a Pair of Quadrupole Lenses

#### 3.3 Pure CSEM Multipoles

To design an iron-less magnet that produces a potential according to equation (3), validity of linear superposition of fields from different pieces of the CSEM makes the question “what is the optimum orientation of the easy axis as function of  $r$  and  $\phi$  to contribute most to a potential described by equation (3)?” (See Figure 2) a reasonable question.

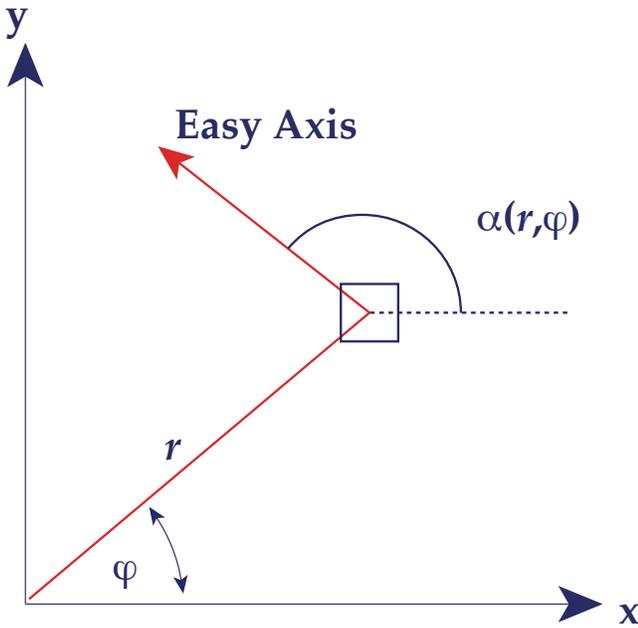


Figure 2 — Optimization of Easy Axis Orientation

The answer to that question<sup>1</sup> is

$$\alpha(r, \phi) = (n + 1)\phi \quad (4)$$

Unfortunately, it would be exceedingly difficult to make material according to that prescription, but the next best thing namely approximating equation (4) by segmentation, gives nearly the same performance. Figure 3 shows schematically a quadrupole designed this way, with the arrows inside the trapezoidal blocks indicating the easy axis orientation in the blocks.

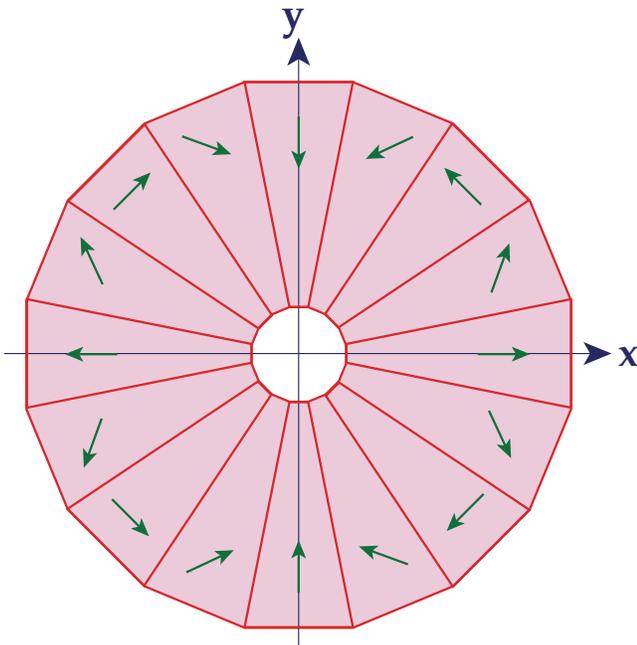


Figure 3 — Pure CSEM Segmented Quadrupole

The field at the aperture radius of such a magnet is given by<sup>2</sup>

$$B(r_1) = B_r \frac{-n}{n-1} \left( \cos \frac{\pi}{M} \right)^n \frac{\sin \left( \frac{n\pi}{M} \right)}{n\pi} \left( 1 - \left( \frac{r_1}{r_2} \right)^{n-1} \right); n > 1 \quad (5a)$$

$$B(r_1) = B_r \frac{\sin \left( \frac{2\pi}{M} \right)}{2\pi} \ln \left( \frac{r_1}{r_2} \right); n = 1 \quad (5b)$$

with  $r_1, r_2$  representing the inside and outside radius of the magnet. and  $M$  = number of blocks in a magnet. In deriving equation (5). it has been assumed that  $\mu_{\parallel} = \mu_{\perp} = \mu$  in equations (1) and (2), and that they are valid over the whole range of values of  $B, H$  occurring in the magnet. particularly down to  $\mu_0 H_{\parallel} = -B(r_1)$ . Notice, in particular, that for  $n = 2, B(r_1)$  can approach  $1.5 B_r$  requiring that the  $B_{\parallel}$  vs  $\mu_0 H_{\parallel}$  curve is a straight line well into the third quadrant. Fortunately, many manufacturers produce materials that satisfy this requirement.

The field strength achievable with this type of quadrupole exceeds that obtainable with a room temperature electromagnet with iron poles. In addition, the pure CSEM quadrupole is exceedingly compact. For that reason, this type of quadrupole is very well suited for use as drift tube quadrupole in fixed energy linear accelerators<sup>3</sup> and in fixed energy beam lines.

### 3.4 Hybrid Multipoles

While methods have been suggested to make quadrupole systems of this type that have adjustable focusing strength<sup>4,5</sup>, their implementation is sufficiently difficult that to the best knowledge of this author, no systems of that kind have been used so far. However, a class of multipole magnets has been developed that uses iron together with CSEM that produces a strong field that is easily adjustable.<sup>6,7</sup>

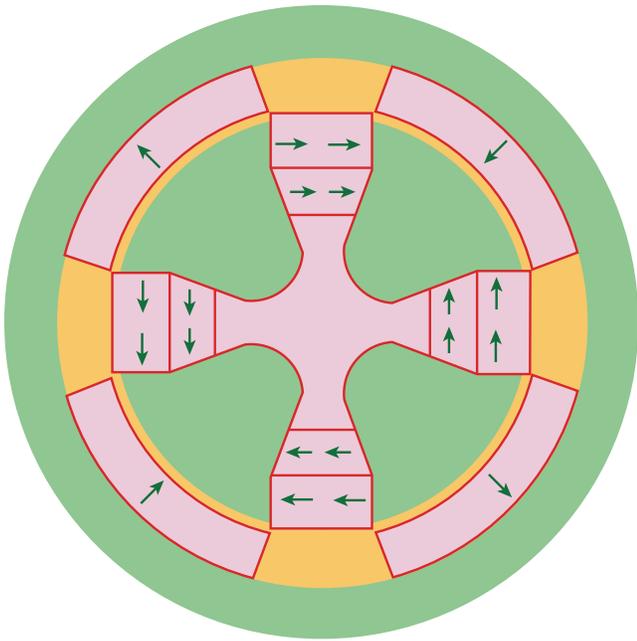


Figure 4 — Adjustable Strength Hybrid Quadrupole

Figure 4 shows a 2D cross section of the quadrupole member of that multipole family. The green areas represent iron and the pink areas with arrows inside them identify CSEM, with the arrow indicating the easy axis orientation inside the block. The outer iron ring has CSEM attached to it, and by rotating that ring with the attached CSEM. One can change the field strength in the aperture region without changing the field distribution. While this hybrid quadrupole is not quite as compact as the pure CSEM quadrupole, and the field strength at the aperture radius is limited (because of iron saturation) to a respectable value of about 1.2-1.4T, the field variability is obviously a great asset. In addition, the field distribution in the aperture region of the hybrid quadrupole is controlled by the iron, and is therefore insensitive to homogeneity of magnetization of the CSEM blocks. The volume integral of the magnetization over each CSBM block is important for equal excitation of all poles. However, an old theory of perturbation effects in iron dominated magnets<sup>8</sup> has been amended to include these effects, and block sorting procedures have been developed to make the resulting field errors insignificant. Two prototype hybrid quadrupoles have been built.<sup>9</sup> Each performed as expected.

It is clear from Figure 4 that multipoles of any order (or even hybrid magnets that produce combinations of multipole fields) can be built with this basic design. While this includes dipoles, for dipoles magnets the design shown in Figure 5 can be advantageous. Very compact fixed field dipole magnets of this basic design have been built by Field Effects Inc. and used successfully by Los Alamos National Laboratory as spectrometer magnets.

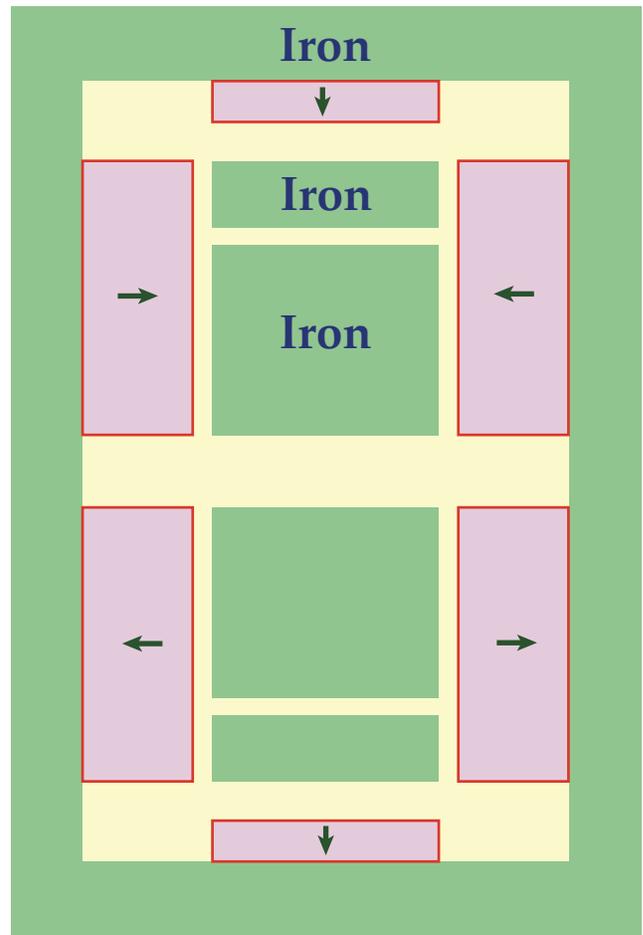


Figure 5 — Hybrid Dipole

## 4. Undulators/Wigglers for Free Electron Lasers and the Generation of Synchrotron Light

### 4.1 Pure CSEM Undulators / Wigglers for Production of Linearly Polarized Light

A very effective method to obtain spontaneous or stimulated electromagnetic radiation from high energy electrons is to expose them to strong static magnetic fields that alternate (spatially) rapidly.

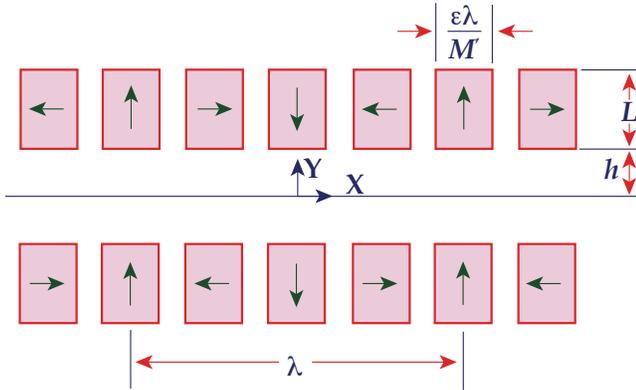


Figure 6 — Pure CSEM Undulator

Figure 6 shows schematically such a device, a pure CSEM undulator/wiggler (U/W). In Figure 6, the electrons travel from the left to the right and “wiggle” in the direction perpendicular to the paper plane. Since it is desirable under most circumstances that the electrons “see” fields that are independent of the coordinate perpendicular to the paper plane, we restrict again the discussion to the 2D aspects of the magnets. The 2D fields produced by the structure shown in Figure 6 is dominated by (see reference 4 for a more complete expression);  $k = 2\pi/\lambda$ ;  $M'$  = number of blocks/period in one array.

$$B_x - iB_y = 2B_r \cos[k(x + iy)] e^{-kh} (1 - e^{-kL}) \frac{\sin\left(\frac{\epsilon\pi}{M}\right)}{\frac{\pi}{M}} \quad (6)$$

The device obviously has to be wide enough for this formula to be applicable, and formulas to assess these 3D effects can also be found in reference 4. Variation of field strength in this device can be achieved by variation of the clear gap between the two arrays of CSEM blocks. It is worth noting that each of the two linear arrays of CSEM in Figure 6 can be obtained by letting the radius of a multipole magnet with fixed radial thickness and period length go to infinity, and equation (6)

can be obtained by this process from the general multipole equation in reference 1.

### 4.2 Hybrid Undulators / Wigglers for Production of Linearly Polarized Light

A disadvantage of the design shown in Figure 6 is the sensitivity of the fields to quality Control in the production of the CSEM blocks. Even though assigning blocks to locations in the arrays according to measured magnetic properties<sup>10,11,12</sup> helps reduce these undesirable effects, the hybrid design shown in Figure 7, using iron in addition to CSEM is much less sensitive to material tolerances.

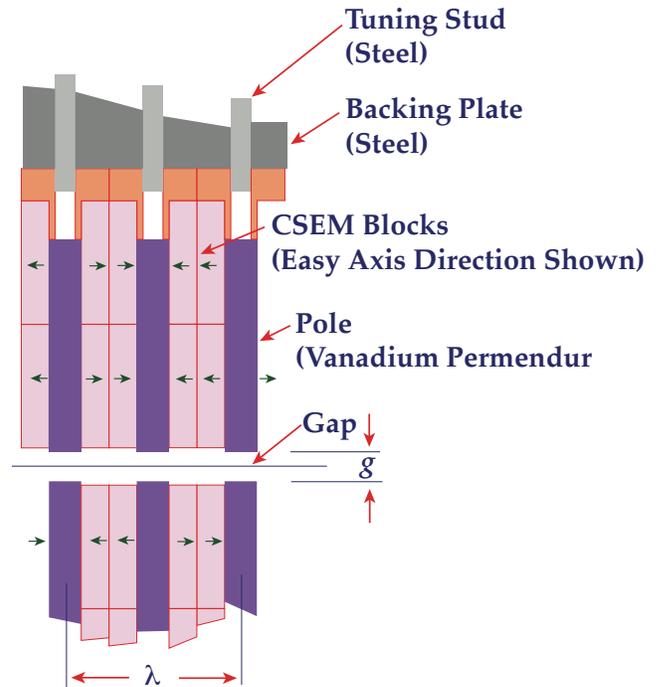


Figure 7 — Hybrid Wiggler/Undulator

In addition, the hybrid can produce stronger fields than the pure CSEM U/W. A 2D computer optimization of the peak field  $B$  in the midplane of a hybrid U/W has been performed for CSEM with  $B_r = .9T$ ,  $\mu_{\parallel} = \mu_{\perp} = 1$ .  $B_{\parallel} = .2B_r$  in the bulk of the CSEM and the saturation characteristics of Vanadium Permendur for a number of different values of the ratio of gap  $g$  divided by U/W period  $\lambda$ <sup>13</sup> within the range  $.07 > g/\lambda \leq .7$ . The results can be well represented by

$$B = 3.33 \exp\left[-\left(5.47 - 1.8 \frac{g}{\lambda}\right) \frac{g}{\lambda}\right] \quad (7)$$

Here, as with the pure CSEM U/W, 3D effects have to be taken into account and corrected to actually get the performance given by (7). These procedures have been developed, but not published yet.

### 4.3 Systems for the Production of Circularly Polarized Light

The U/W described above produce linearly polarized light in the forward direction. For the production of circularly polarized light one can use a helical U/W.

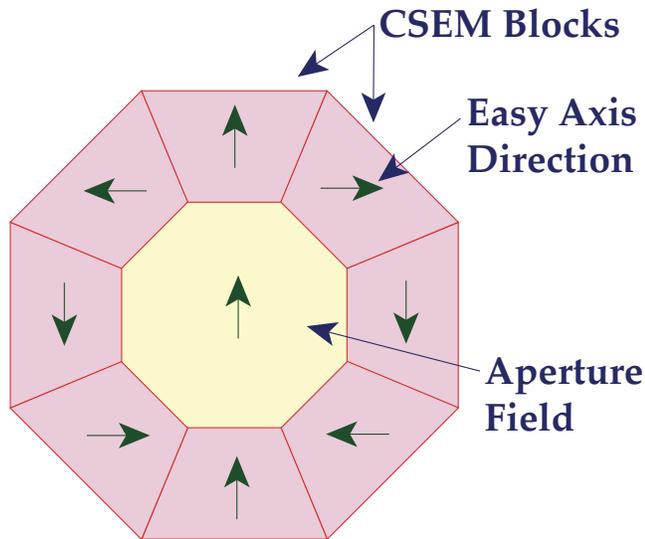


Figure 8 — Pure CSEM Dipole

Figure 8 shows the 2D cross-section of a pure CSEM dipole. By making axially short segmented dipoles and rotating each short dipole by a fixed angle relative to the previous dipole, one can obtain the desired helical field. The expected performance of such a helical U/W is given in reference 4. Another way to produce circularly polarized light with permanent magnets undulators is schematically shown in Figure 9:<sup>4</sup>

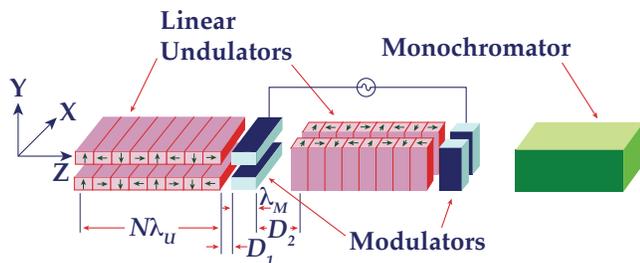


Figure 9 — System for Production of Elliptically Polarized Synchrotron Light

The two undulators produce two linearly polarized wave trains. By passing them through a monochromator of sufficiently narrow bandwidth, each wave train is lengthened, thus leading to significant overlap of the wave trains, and with it elliptically polarized light. By adjusting the electron trajectory length between the two undulators with the modulator, the phase shift between the two wave trains can be adjusted, and with it the polarization properties of the light emerging from the monochromator.

## 5. Conclusions

Permanent magnet systems have been developed that are used very successfully in accelerators and as sources of synchrotron radiation in electron storage rings. The main advantage of these systems is the possibility to obtain, under some circumstances, performance Characteristics that can not be obtained with any other technology. One unique advantage of pure CSEM devices not mentioned above is the possibility of “immersing” a pure CSEM magnet in the field of another magnet, with essentially linear superposition of the fields produced by the two magnets. Examples are, for instance, a pure CSEM U/W inside a quadrupole (that provides focusing for the particles), and a pure CSEM quadrupole inside a solenoid. Under other circumstances, the advantage may be merely one of economy or convenience. A recently published proposal<sup>5</sup> to build an electron storage ring entirely with permanent magnets is an indication that this technology will be used even more frequently in the future.

## Publishing History

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