
Permanent Multipole Magnets with Adjustable Strength

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Abstract

Preceded by a short discussion of the motives for using permanent magnets in accelerators, a new type of permanent magnet for use in accelerators is presented. The basic design and most important properties of a quadrupole will be described that uses both steel and permanent magnet material. The field gradient produced by this magnet can be adjusted without changing any other aspect of the field produced by this quadrupole. The generalization of this concept to produce other multipole fields, or combination of multipole fields, will also be presented.

1. Introduction

Permanent magnet systems that use no steel, but only Charge Sheet Equivalent Material (CSEM)^{1,2} have a number of unique advantages compared to hybrid systems that use both CSEM and steel. The most important of these is probably the possibility to embed the permanent magnet into an external field, with linear superposition of fields everywhere. When these properties are not needed, it can be advantageous to use hybrid magnets. They are clearly not as critically sensitive to material properties, and it is now possible to build magnets whose strength can be adjusted without affecting the field distribution. Since the development of the original conceptual design of a variable strength hybrid quadrupole,³ a considerable amount of work has been done on this quadrupole. The major results of that work, and the related properties of this hybrid quadrupole, will be summarized below. While the subject of this discussion is the quadrupole, it will be clear that practically all of this information is applicable, after a suitable “translation,” to other multipole magnets, or combined function magnets.

Since it is the author’s experience that the major reasons for using permanent magnets are not always as clearly understood as they ought to be, a short discussion of this topic will precede the listing of the quadrupole properties.

2. Motivations For Using Permanent Magnets

When one changes all linear dimensions (L) of an electromagnet while keeping the field strength at equivalent locations fixed, the current density in the coils varies like $1/L$, and consequently the power density in a conventional coil varies like $1/L^2$. From this it follows directly that below a certain critical size of a magnet, a superconducting coil will go normal, or a conventional coil will become “un-coolable.” If, to avoid that, one increases the coil dimensions, a reduction of field strength will invariably follow. This is true both in magnets that use no steel as well as in magnets that use steel that is close to saturation. Since permanent magnets clearly do not suffer from this limitation, there will always be a limiting linear dimension below which the performance of a permanent magnet will exceed that of an electromagnet, thus increasing the parameter space that can be used by the particle-optical system designer. The above mentioned possibility of immersing a pure CSEM magnet in an external field falls into the same category of permanent magnets being able, under some circumstances, to produce fields that can not be obtained by any other means.

The second major reason for preferring permanent magnets to electromagnets can be the absence of a sometimes stiff bill for electric power, power supplies with the associated cables, and cooling. This economic argument can be a very strong one and it is often valid when one is close to the size limit where the performance of the permanent magnet exceeds that of the electromagnet.

3. Variable Strength Quadrupole Geometry, Operation, and Properties

Figure 1 shows a schematic two dimensional (2D) cross-section of a quadrupole, with the green areas identifying soft iron, and the arrows indicating the direction of the easy axes of the CSEM. The preferred CSEM is Rare Earth Cobalt (REC) if extreme strength and compactness are important, otherwise those oriented ferrites can be used that have a $B(H)$ curve that is straight at least in all of the second quadrant. To change the strength of the field in the aperture region, the outer steel ring, with the attached CSEM, is rotated about the center of magnet, while the four poles with their CSEM stay fixed. Depending on the details of a specific design, 90° rotation of the outer ring from the position shown in Figure 1 reduces the field to any desired fraction of the maximum value. If field variability is not necessary, the outer circular arc of each pole can be replaced by a straight line that encloses $+45^\circ$ with the x - and y -axes, or two straight lines parallel to the x - and y -axes. The outer ring would be replaced by a square steel box, with the CSEM between the poles and the square yoke touching both surfaces.

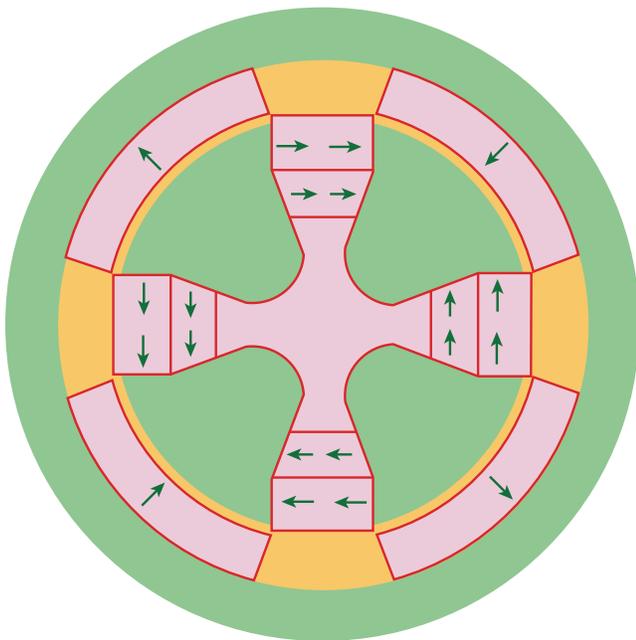


Figure 1 — Schematic cross-section of a variable strength permanent quadrupole magnet.

Using REC and Vanadium Permendur, a field strength of 1.3-1.4 T can be achieved on the aperture radius that touches the poles. The ratio of the outside diameter of the quadrupole to the aperture diameter is about 4 to 6 for moderate to high aperture fields, and about 10 to 15 for very high aperture fields.

The field in the aperture region is controlled by the steel configuration as in a conventional electromagnet. This

means that a great amount of know-how about electro-magnets (like necessary pole width to achieve a given field quality, saturation effects, etc.) is directly applicable to the design of hybrid quadrupoles. As in an electro-magnet, the allowed harmonics are $n = 2, 6, 10, 14, \dots$. This is true even when the outer ring has been rotated, since that does not violate invariance of the total geometry under rotation by 90° .

When the outer ring, with the attached CSEM is rotated, two mechanisms exist that can produce a skew quadrupole component:

- A) The field distribution in the outer part of the steel pole can be quite asymmetric, producing, through iron saturation, a skew quadrupole component. It is, however, possible to design the quadrupole such that the field in the outer part of the steel pole is so weak that the associated large permeability reduces the resulting skew quadrupole strength beyond any point of concern.
- B) The CSEM generates fields directly in the vacuum region between the poles that violate the regular field symmetry. The thusly generated skew quadrupole is, however, so small (typically less than 10^{-4} of the strength of the regular quadrupole) that compensation is not necessary, though easily achievable.

As was outlined in section 2, permanent magnet structures are very advantageous when they are small. The converse is also true: The volume V of CSEM in a hybrid quadrupole depends on the gradient B' , aperture radius r , and remanent field B_r in roughly the following way:

$$V \approx B'^2 \frac{r^4}{B_r^2} \quad (1)$$

indicating that the cost of the material becomes prohibitive when r gets too large.

4. Design Procedure of Quadrupole With Given Aperture and Strength

It is clear from Figure 1 that there are so many parameters involved in the design of a hybrid quadrupole that it would be foolish to try to design it from the beginning with a big computer code like PANDIRA.⁴ For that reason a very short code has been written that is used to design, with analytical formulas, hybrid quadrupoles. The strength of a quadrupole that has been designed with this codes is typically within about 5% of the design goal, and what follows is a description of the major thoughts that went into this design code and procedure.

Figure 2 shows, schematically, a 45° slice of a hybrid quadrupole. The half width of the pole is designed such

that the required field quality can be achieved in the aperture. Since it is particularly important for quadrupoles that the pole is not wider than absolutely necessary, it is usually assumed that the pole has proper shims.

When designing a strong quadrupole, it is very important to place CSEM as close as possible to the aperture in order to reduce field reduction through saturation of the steel. The coordinate x_2 is therefore set so small that $(H_y)_{\text{Max}}$ in the first CSEM block is only a little smaller than the largest value of H permissible for the CSEM without getting into the nonlinear part of the $B(H)$ curve. In a weak quadrupole, block 1 may be absent in order to reduce total manufacturing cost.

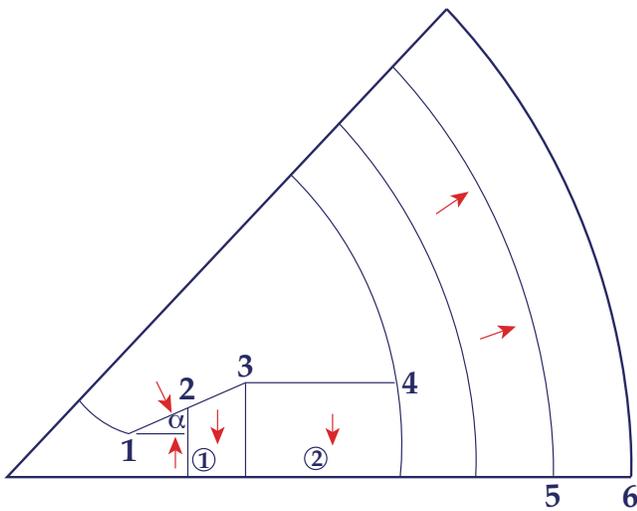


Figure 2 — Schematic cross-section of a 45° slice of a variable strength permanent quadrupole magnet.

The value of y_3 is usually set so that the total amount of CSEM is minimized. Contrary to frequently described general rules about CSEM volume minimization, this minimization is achieved when H_y in CSEM block 2 is somewhat less than half the coercive field H_c . Since one is close to the optimum with this general kind of choice, the exact value of y_3 is not very critical.

The CSEM ring is, for economic reasons, assembled from uniformly magnetized blocks. Their thickness in the radial direction, and the gap between that CSEM and the steel pole, are determined by the desired range of strength adjustment, and, usually, also to minimize the total amount of CSEM. In this case, H in the CSEM should be (again contrary to often stated general rules) somewhat larger than $0.5 H_c$. Under some circumstances, it is advantageous to split the CSEM ring into two rings, with the inner ring permanently attached to the steel pole. Details of this special design will be discussed in a future paper.

The distance $x_4 - x_3$ is used to give the total magnet the desired strength, and $x_6 - x_5$ is chosen so that saturation of the steel ring is sufficiently small.

The angle α has a pronounced effect on both saturation of the steel in the region behind the pole, the overall size of the magnet, and the total amount of CSEM. Since the relative importance of these qualities is usually very different for different magnets, the optimum value of α is selected by “brain-judgement” from a table of designs with different values of α .

For most applications, it is desirable to have a field clamp at each end of the quadrupole, and it is advantageous to have CSEM between the poles and the clamp. This is particularly true for strong quadrupoles. If one wants to minimize the total amount of CSEM, the CSEM at the ends should have a value of H that is close to $0.5 H_c$.

In order to reduce the net torque that has to be applied to the poles, it can be advantageous to break up the outer steel ring with the attached CSEM into two or three sections lengthwise, neighboring sections being rotated in opposite directions for strength adjustments.

5. Field Errors, Tolerances

The thorough understanding of tolerances and the resulting field errors is clearly very important if one has to build a conventional quadrupole with good field quality. These topics are even more important for the hybrid quadrupoles discussed here since the excitation of the poles is accomplished with CSEM instead of coils. An investigation of these tolerance effects shows that it is inexpensive and fairly easy to avoid or correct such field errors, but only if one has a good understanding of the theory that describes them. The following is a summary of some of the results of a theory that deals only with 2D effects. The reason for that restriction is the fact that in the overwhelming majority of cases, one is interested in, and measures, only line integrals of fields, making a 3D treatment unnecessary.

The effects of perturbations of the steel poles (displacement, machining errors, etc.) have been described elsewhere⁵ so that only effects associated with CSEM tolerances need be described here. These perturbations lead to two kinds of field errors: those that are associated with, or caused by, perturbations of the scalar potentials of the poles, and that I refer to as indirect field errors; and field errors that are not associated with scalar potential changes of the poles, referred to as direct field errors.

Indirect field errors can be characterized by a unique pattern of harmonics⁵ that is geometry-independent, and there are three combinations of pole potential perturbations that are of interest.

Direct field errors can be characterized by two unique, geometry-independent and analytically calculable, patterns of harmonics, and in the whole quadrupole there

are 8 strength factors associated with these two patterns.

By measuring the harmonic content of the field distribution, it is *very* easy to extract all relevant strength parameters, and to apply corrective measures.

Computer runs have been made to evaluate the field errors that are caused by some specific CSEM errors in a specific, but typical, geometry. As mentioned in Section 3, the skew quadrupole component caused by rotation of the steel-CSEM ring is typically less than 10^{-4} of the regular quadrupole field, and is easily correctable if necessary. Also investigated were the effects of a 5° error in easy axis orientation of block 1 in Figure 2, and of a gap between the top of block 1 and the steel. The size of the gap was 10% of the aperture radius, and all the direct field error harmonics produced by the above perturbations were less than 0.1% of the quadrupole strength at the aperture radius. The effects of equivalent errors on block 2 were considerably smaller. The indirect field errors were smaller than 3% and are very easily correctable. Indirect field errors caused by errors in magnetization strength can be easily avoided by measuring the dipole moment of every block and then placing blocks appropriately in the magnet.

6. Other Magnets

Addressing first the design of dipoles, a statement made in Reference 3 needs to be corrected; adjustable strength dipoles can be made with essentially the same strength as conventional dipoles, i.e., 2T are easily reachable, even with hard ferrites (if large overall size can be tolerated). In the case of a dipole, strength adjustment by a method other than by rotating an outer ring may be preferable. This dipole configuration is schematically indicated in Figure 3. Strength adjustment can be achieved by movement of blocks A toward, or away from, the midplane. Both methods of strength adjustment are obviously also applicable to combined function magnets, i.e., magnets that have, in addition to a dipole component, higher multipole components present.

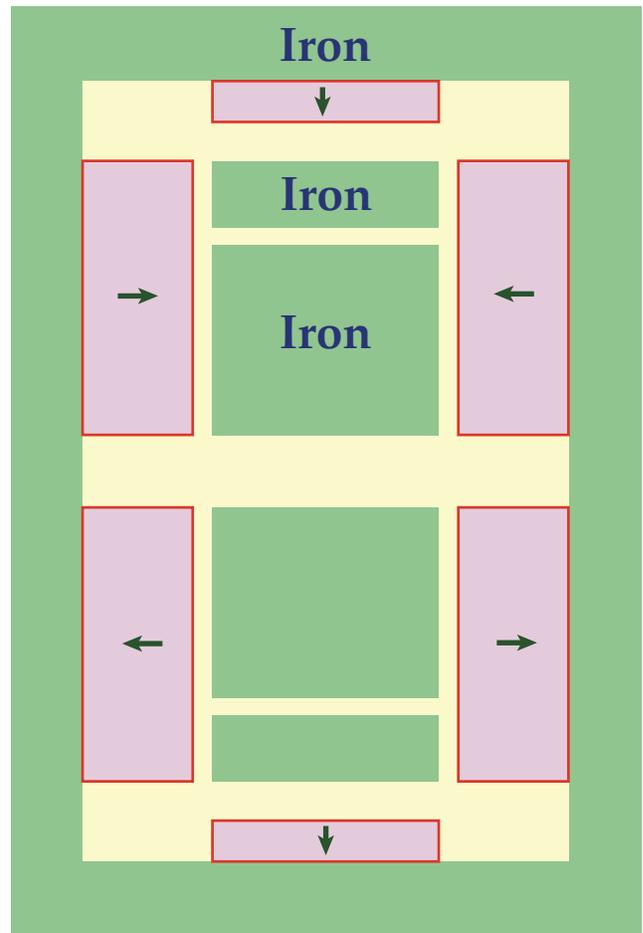


Figure 3 — Schematic cross-section of a variable strength permanent dipole magnet.

Higher order adjustable strength permanent multipole magnets, like sextupoles, octupoles, etc., can obviously also be made with the same basic concept evident in the quadrupole. The described design procedure of such a magnet would be nearly identical to that of a quadrupole. The tolerance theory would be patterned after that theory for the quadrupole. It would not be more complex, only lengthier.

Publishing History

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⁴ PANDIRA is a code that allows evaluation of 2D permanent magnets and was developed by Klaus Halbach and Ron F. Holsinger.

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