

# The Inductrack Approach to Magnetic Levitation

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## Abstract

Concepts developed during research on passive magnetic bearing systems at the Lawrence Livermore National Laboratory gave rise to a new approach to magnetic levitation, the Inductrack. A passive induced-current system employing permanent magnets on the moving vehicle, the Inductrack maximizes levitation forces by a combination of two elements. First, the permanent magnets on the vehicle are arranged in a "Halbach array," a magnet configuration that optimally produces a periodic magnetic field below the array, while canceling the field above the array. Second, the "track" is made up of close-packed shorted electrical circuits. These circuits couple optimally to the magnetic field of the Halbach array. As a result, levitating forces of order 40 metric tonnes per square meter of Halbach array can be generated, using NdFeB magnets whose weight is a few percent of the levitated weight. Being an induced-current system, the levitation requires motion of the vehicle above a low "transition speed." For maglev applications this speed is a few kilometers per hour - walking speed. At rest or in the station auxiliary wheels are needed. The Inductrack is thus "fail-safe," that is, drive system failure would only result in the vehicle slowing down and finally settling on its auxiliary wheels. On the basis of theoretical analyses a small model vehicle and a 20-meter-long track was built and tested at speeds of order 12 meters per second. A second model, designed to achieve 10-g acceleration levels and much higher speeds, is under construction under NASA sponsorship, en route to the design of maglev-based launchers for rockets. Some of the presently perceived practical problems of implementing full-scale maglev systems based on the Inductrack concept will be discussed.

## 1 Introduction

Thanks to several decades of development work the technical feasibility of magnetically

levitated high-speed trains has been amply demonstrated. These demonstrations include the Transrapid trains in Germany, employing electromagnetic attraction, and the Japanese development of induced-current systems employing superconducting magnets. However, even though both these systems are technically viable, neither one has as yet been implemented as a commercially operating transportation link. Among the reasons why such deployment has not as yet occurred, technical complexity and projected cost of these systems can be cited. Another reason may be the perceived hazards associated with failure modes of either system: Failure of the electrical systems on the German system could lead to loss of stable levitation, as could failure of the cryogenic or electrical system on the Japanese train. Good engineering practice and system redundancy can in both of these cases minimize such hazards, but not without an increase in cost and complexity.

At the Lawrence Livermore National Laboratory we have been exploring another type of maglev system, dubbed the Inductrack [1], that represents a simpler approach than either the electromagnetic Transrapid system or those systems employing superconducting magnets. The Inductrack concept grew out of work at the Laboratory on passive magnetic bearings [2,3] the stability of which was derived from electrodynamic interaction between a moving array of permanent magnets and a close-packed array of shorted electrical circuits. Unrolled conceptually to form a linear track, this combination of elements overcomes past objections to the use of permanent magnets for maglev systems. These objections arise from the perceived inadequacy of permanent-magnet systems to produce adequate levitation forces in an induced-current system, as compared to superconducting magnets with their much higher magnetic fields.

In the Inductrack the cited objection is overcome by a combination of two elements. The first of these is the use of the Halbach-array [4] configuration for the permanent magnets, located on the moving vehicle. The Halbach

array represents an optimally efficient way to utilize permanent-magnet material to produce a sinusoidally periodic and spatially concentrated magnetic field below the array, while at the same time canceling the field above the array. The second key element of the Inductrack is the “track” itself. To optimize both the inductive coupling between the moving Halbach arrays and the track, it consists of a close-packed array of shorted circuits. Such an arrangement maximizes the active area that generates the levitating force.

As a result of these two optimizing factors, when using high-field magnetic material, such as NdFeB, with remanent fields in excess of 1.2 Tesla, useful levitating forces of order 40 metric tonnes per square meter of magnet array area can be generated, requiring magnets weighing as little as 2 percent of the levitated weight.

Since it is an induced-current repulsive-force system, levitation is only achieved in the Inductrack when the vehicle is moving. However the “transition speed” (here defined as the speed when the levitation force has risen to half its asymptotic value) is typically only a few kilometers per hour - walking speeds. In the station or at speeds below the transition speed the Inductrack would employ auxiliary wheels. The low value of the transition speed, plus the use of permanent magnets and auxiliary wheels insures the “fail-safe” nature of the Inductrack system, for example, if its drive system should fail.

Another valuable attribute of the Inductrack is that its operation can be predicted accurately on the basis of theoretical analysis. This analysis resulted in equations that accurately depict the levitation and drag forces, including issues of stability. These equations can then be employed to facilitate the design of practical systems and for the estimation of costs. They were also used early on in the studies to design a small-scale working model of the Inductrack, the performance of which agreed closely with the theory. At the present time the theory is being used to design a new small-scale model at the Laboratory, under NASA sponsorship, en route to using the Inductrack as a means for launching rockets into space.

## 2 Summary of Theoretical Analyses

The theoretical analysis of the Inductrack levitation and drag forces can be performed using Maxwell's equations and standard electrical circuit theory. The starting point for the analysis is the equation for the voltage and

currents induced in a typical track circuit, as follows:

$$V = L \frac{dI}{dT} + RI = \omega\phi_0 \cos(\omega t) \quad (1)$$

Here V (volts) is the induced voltage, I (amps) is the induced current, L(hy) is the inductance of the circuit (self-inductance plus the effect of inductive coupling to adjacent circuits), R (ohms) is the resistance of a circuit, and  $\phi_0$  (Tesla-m<sup>2</sup>) is the peak flux linked by the circuit owing to the passage of the Halbach array above the circuit. The frequency,  $\omega$  (radians/sec.) is defined by the wavelength,  $\lambda$  (m.) of the Halbach array and its velocity, v (m/sec.) relative to the track through the relationship  $\omega = kv$ , where  $k = 2\pi/\lambda$ .

The steady-state solution of this equation is:

$$I(t) = \frac{\phi_0}{L} \left[ \frac{1}{1 + (R/\omega L)^2} \right] \{ \sin(\omega t) + (R/\omega L) \cos(\omega t) \} \quad (2)$$

As can be seen from (2), in the limit  $\omega \gg R/L$  (i.e. for velocities that are substantially higher than the transition velocity), the phase of the induced current is retarded by nearly 90° with respect to the induced voltage. This shift in phase has the effect of maximizing the lift relative to the drag forces, leading to a levitation force that approaches a constant value and a drag force that varies inversely with velocity.

A schematic representation of the Inductrack, showing a Halbach array moving above the close-packed conductors of the track is shown in Fig. 1. The orientation of the magnetization of the individual bars as shown is such as to concentrate the field lines below the array while nearly canceling the field above the array. The magnetic field lines below the array are also shown schematically in the figure. The associated magnetic flux links with the circuits to produce currents that then interact back with the horizontal component of the field to produce the levitating force.

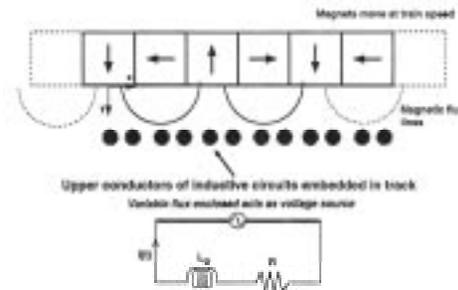


Fig. 1: Schematic diagram of Inductrack concept.

## 2.1 Calculation of the Levitation and Drag Forces

To calculate the lift and drag forces we turn to the theory of the Halbach array as derived by its inventor, Klaus Halbach [4]. The horizontal (x) and vertical (y) components of the magnetic field from a planar Halbach array are given by the expressions:

$$B_x = B_0 \sin(kx) \exp[-k(y_1 - y)] \quad (3)$$

$$B_y = B_0 \cos(kx) \exp[-k(y_1 - y)] \quad (4)$$

Here  $y_1$  is the vertical distance between the lower surface of the Halbach array and the centroid of the upper conductors of the track.  $B_0$  is the peak strength of the magnetic field at the surface of the Halbach array, given by the expression:

$$B_0 = B_r [1 - \exp(-kd)] \frac{\sin(\pi/M)}{\pi/M} \quad (5)$$

In this expression  $B_r$  (Tesla) is the remanent magnetic field of the permanent magnet material,  $d$  (m.) is the thickness of the Halbach array magnets, and  $M$  is the number of magnet bars per wavelength in the Halbach array. For the configuration shown in Fig. 1,  $M = 4$ . For 48-grade NdFeB magnet material, for which  $B_r = 1.41$  Tesla, and with  $d = \lambda/4$  (i.e., magnet bars with a square cross-section),  $B_0 = 1.0$  Tesla.

To calculate the levitation and drag forces we first consider a track composed of close-packed shorted circuits in the form of rectangular "window frames" with a transverse width,  $w$  (m.), and with a vertical spacing between the upper and lower conductors of  $h$  (m.), so that the area of each circuit is  $wh$  ( $m^2$ ). We will assume that the thickness of the circuit conductors,  $d_c$  (m.), is much less than a wavelength, specifically that  $d_c \ll k^{-1}$ . Integrating the expression for  $B_x$  over the area of the circuit gives an expression for the flux linked by each circuit:

$$\phi = \frac{wB_0}{k} \exp(-ky_1) \sin(kx) [1 - \exp(-kh)] \quad (6)$$

In this expression the exponential term in brackets,  $\exp(-kh)$ , representing a term correcting the flux linkage for field lines passing below the circuits, is normally very small compared to unity and will be ignored in what follows. Inserting this expression for the

flux into (2), and substituting  $x = vt$ , we find an equation for the induced current (in the transverse,  $z$ , direction) in terms of the track and magnet parameters:

$$I_z(t) = \frac{\lambda B_0 w}{2\pi L} \left[ \frac{1}{1 + (R/\omega L)^2} \right] \exp(-ky_1) \{\sin(kx) + (R/\omega L) \cos(kx)\} \text{ amps/circuit} \quad (7)$$

This current then interacts with the horizontal and vertical components of the magnetic field to produce the levitation and drag forces:

$$F_y = I_z B_x w, \quad F_x = I_z B_y w \text{ Newtons/circuit} \quad (8)$$

Averaging these expressions over the wavelength one finds the average levitation and drag forces, given by the expressions:

$$\langle F_y \rangle = \frac{B_0^2 w^2}{2kL} \left[ \frac{1}{1 + (R/\omega L)^2} \right] \exp(-2ky_1) \text{ N/circuit} \quad (10)$$

$$\langle F_x \rangle = \frac{B_0^2 w^2}{2kL} \left[ \frac{(R/\omega L)}{1 + (R/\omega L)^2} \right] \exp(-2ky_1) \text{ N/circuit} \quad (11)$$

The Lift/Drag ratio can be obtained immediately by dividing (10) by (11), finding:

$$\frac{\text{Lift}}{\text{Drag}} = \frac{\langle F_y \rangle}{\langle F_x \rangle} = \frac{\omega L}{R} = \frac{2\pi v}{\lambda} \left[ \frac{L}{R} \right] \quad (12)$$

As noted previously, the Lift/Drag ratio increases linearly with the velocity.

This expression can be used to evaluate the levitation efficiency, that is the Newtons of levitating force per Watt of power dissipated in the track. Since the average power,  $\langle P \rangle$  (Watts), dissipated per circuit is given by the product  $v \langle F_x \rangle$ , we find from (12):

$$K = \frac{\langle F_y \rangle}{\langle P \rangle} = \frac{2\pi}{\lambda} \left[ \frac{L}{R} \right] \text{ Newtons/Watt} \quad (13)$$

Comparing (13) for the power efficiency and (10) for the levitation force, we see that for any given circuit resistance,  $R$ , we can obtain any desired degree of efficiency by adding loading inductance to the circuits, but necessarily at the expense of reducing the lifting force per circuit.

These equations can also be related to the transition velocity, which we define as the

velocity at which the levitation force has risen to one-half of its limiting value at high speeds. From (10, 11, and 13) we see that the transition velocity is related to the levitation efficiency, since it is that velocity where the product  $Kv = 1.0$ . For example, if  $K = 1.0$  Newton/Watt (a typical value in the absence of extra inductive loading), the transition velocity,  $v_t$ , is 1.0 m/sec. or 3.6 km/hr, a slow walking speed. With inductive loading it can, of course, be made even lower. Note also, from (10), that at a velocity that is only 2 times the transition velocity the levitation force has already risen to 80 percent of its limiting value.

To find the levitation force per unit of area of Halbach array at velocities that are large compared to the transition velocity we evaluate (10) in that limit and note that the number of circuits per meter of track length is  $(1/d_c)$ , and the area per meter of track length is equal to  $w$ , so that:

$$\frac{\Sigma < F_y >}{A} = \frac{B_0^2 w}{2kLd_c} \exp(-2ky_1) \text{ Newtons/m}^2 \quad (14)$$

In the case that no inductive loading is added to the circuits, the inductive term,  $L$ , is composed of the self-inductance of each circuit as modified by the mutual inductance of the adjacent circuits, forming what we call a "distributed inductance," as given by the following term derived from the theory:

$$L_d = \frac{\mu_0 P_c}{2kd_c} \text{ Henrys} \quad (15)$$

In this expression  $P_{c(m)}$  is the perimeter of the circuits (top, two sides, and bottom conductors). Inserting this expression into (14) gives for the levitation force per unit area the expression:

$$\frac{\Sigma < F_y >}{A} = \frac{B_0^2}{\mu_0} \left[ \frac{w}{P_c} \right] \exp(-2ky_1) \text{ Newtons/m}^2 \quad (16)$$

This equation deserves some discussion, as it demonstrates an important feature of the Inductrack with respect to the magnitude of the levitating force. Note that the quantity  $B_0^2 \exp(-2ky_1)$  is equal to the square of the peak magnetic field of the Halbach array as evaluated at the location of the upper track conductor. Noting that the value of the electromagnetic stress tensor is equal to  $B^2/2\mu_0$ , it can be seen that, apart from the term representing the ratio of the width of the window-frame circuit to its perimeter (of order

1/3), the levitation force is twice the value one would expect to derive from the value of the stress tensor associated with the peak field of the Halbach array at the distance  $y_1$  from the centroid of the upper conductor of the circuit. The extra factor of two comes from the doubling of the peak incident magnetic field at the conductor from the "image" effect associated with the close-packed nature of the track circuits. This field doubling would itself give a factor of four increase in the force, but it is then diminished by a factor of two by the spatial averaging, yielding a net factor of two increase. For a  $B_0$  value of 1.0 Tesla, the term  $B_0^2/\mu_0$  is equal to 80 metric tonnes force per square meter. As will be later discussed, achieving levitation forces approaching one-half this value should be possible in practical cases.

## 2.2 Optimization of the Halbach Array Parameters

The availability of an analytic theory of the Halbach array and its use in the Inductrack permits an analysis of the optimum parameters of the magnet system in terms of minimizing the magnet weight relative to the levitated weight. First, from (5) (for the magnetic field of the Halbach array) the variation of  $B_0$  with magnet thickness,  $d$ , can be determined. Since the weight of the magnets is proportional to  $d$ , an equation optimizing  $d$  can be determined by differentiation. From this one finds that the optimum magnet thickness is equal to  $0.2 \lambda$ , close to the value of  $0.25 \lambda$  previously assumed. Next, given the desired value of  $y_1$ , the levitation height to the centroid of the conductor, the wavelength that again optimizes the magnet weight can be found. This optimum value is given by the equation:

$$\lambda_{\text{optimum}} = 4\pi y_1 \text{ (m.)} \quad (17)$$

When these optimized values are inserted into the levitation equations, for an example case where the levitation height is 3.0 cm., and NdFeB magnets are used, the ratio of levitated to magnet weight for a flat-track (see section 3) case was found to approach 50:1. Of course, if inductive loading is used, or if the window-frame track is used, lower values will be found. What values are acceptable will depend on the economic trade-offs involved.

## 3 Track Circuit Configurations and their Fabrication

We have thus far mainly discussed a track

circuits in the form of rectangles, and have not considered either alternative configurations or fabrication methods. An alternative track-circuit configuration is the "flat-track" form. It is one in which the "circuits" are made up of a planar array of conductors, shorted together at their ends. One advantage of this configuration over the window-frame form can be seen from (16), which can be shown to apply to the flat-track configuration as well as the window-frame one. However in the flat-track case the term  $(w/P_c)$  is nearly equal to 1.0, so that nearly the theoretical maximum levitation force associated with the magnetic field incident at the track can be achieved. Along with this comes the possibility of constructing the track by laminating thin anodized sheets of aluminum, or by laminating thin plastic sheets on which have been deposited a pattern of copper (or aluminum) conductors in the form of thin strips shorted at their ends. Using thin strip conductors and laminating them to form the close-packed array reduces parasitic eddy currents that would otherwise compromise the efficiency of the levitation. Figure 2 is a schematic representation of this kind of track configuration.

For those cases where the window-frame construction is preferred (for example if inductive loading is to be used), one could either form the circuits by winding multi-turn coils of small-diameter insulated wire (the construction which we employed in the model that was built [1]), or, again, use a pattern of shorted circuits deposited on thin plastic sheets that are laminated and stacked to form the track. We have not as yet employed these alternative forms for the track in our models, but believe they are worthy of serious consideration.

Depending on the form of the track circuits that is chosen one can evaluate the Lift/Drage ratio using (12). To illustrate the numerical values involved Fig. 3 is a plot of L/D as calculated for two examples. The first is a window-frame track with no added inductive loading. The second, higher L/D, example is one in which inductive loading (through ferrite elements) has been added to increase K to the value 3.0. As can be seen from the figure, at maglev train speeds the L/D ratio for the first case is of order 200:1, to be contrasted with typical values of 25:1 for the wing of a jet airplane. An L/D ratio of 200:1 at a speed of 500 km/hr corresponds to a requirement of less than 300 kilowatts of power required to levitate a train car weighing 40,000 kilograms. Such a car would be estimated to require 8 megawatts of drive power to overcome aerodynamic drag at 500 km/hr, so that levitation would require less than 4 percent of the power needed to overcome air resistance at that speed. Also shown on Fig. 3 is the L/D ratio as calculated from engineering

rails systems at a speed of 250 km/hr. However such mechanically based systems both are limited as to their maximum speed and have L/D ratios do not increase linearly with velocity, as is the case with the Inductrack. Shown also is the L/D ratio

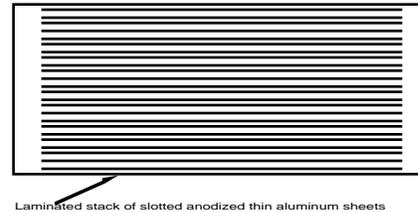


Fig. 2: Example of a possible fabrication technique for a "flat-track" version of the Inductrack.

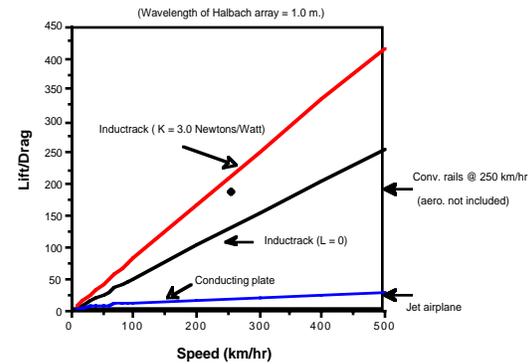


Fig. 3: Comparative Lift/Drage ratios as a function of speed.

calculated for a case where the Inductrack circuits are replaced by a sheet conductor (copper). For this case not only is the L/D much lower, being generally even lower than that for a jet airplane wing, but it also only increases as the square-root of the velocity.

## 4 Model Tests: The NASA Model Launcher Project

No full-scale model of the Inductrack has been built. However, as described in ref. 1, a small-scale model has been built and operated to prove out the theory. This first model was electromagnetically launched on a 5 meter launching section of the track at speeds of order 12 m/sec., down a 15 meter passive track, on which it levitated stably and came to a halt at the end of the track. At the present time a second model, funded by a contract from NASA, is being built to investigate another application for the Inductrack: launching rockets. This model is intended to reach much higher speeds (in excess of 100 meters per second), achieved by 10-g-level electromagnetic acceleration using special drive coils interleaved with the levitation

coils. These drive coils are configured in such a way that they do not couple to the levitation coils (and vice-versa), thereby allowing the use of high-power pulse circuitry to achieve the high acceleration rates without adversely affecting the levitation.

## 5 Summary

Some aspects of the theory of the Inductrack maglev concept and its implementation in model tests have been discussed. Such issues as stability, oscillation damping for rider comfort, and construction costs have been considered theoretically, but were not discussed in this paper. However, the basic simplicity of the Inductrack augers well for its eventual development as a practical high-speed maglev system, as well as for other possible applications, including the electromagnetic launching of rockets, the transport of freight, and urban "people mover" systems.

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