Core Collisional Ion Upscattering and Loss Time

By Robert W. Bussard, February 19, 1991, EMC2-1090-03

1. Introduction

Energy exchange and upscattering in ion-ion collisions as they pass through the core of a Polywell/HEPS device have been analyzed previously by Lovberg[1] and Bussard.[2] One result of these studies is a simple equation describing the energy dispersion introduced over a time t_c by such collisions. This is given by[3]

$$\langle E_d(t_c)\rangle = \pi\sqrt{2nvt_c}\,(Ze)^2 \quad (1)$$

Here $<E_d(t_c)>$ is the total energy spread introduced in an ion population by small angle collisions in the core, after repeated passes through the core by the subject ions. This spread both increases and decreases the energy of ions from their initially-monoenergetic level at the well center. Taking δE as the mean up *or* down dispersion of energy from in-core scattering collisions, gives $2\ \delta E = \pm <E_d(t_c)>$. Defining a function $f = (\delta v/v)$ as the fractional up or down spreading in velocity space gives $(\delta E/E) = 2f$, so that $\delta E = 2fE$. With this, equation (1) becomes

$$efE = \pi\sqrt{2nvt_c}\,(Ze)^2 \quad (2)$$

2. Upscattering Time

This can be solved for t_c as the time required to yield an energy up scatter of $2fE$ or a fractional velocity upscatter of f. In order for this to be correct, the product (nv) must be the integrated average

$$\langle nv\rangle = \frac{1}{R}\int_0^R n(r)v(r)\,dr \quad (2b)$$

over the complete transit path of the ion. This is a measure of the scattering center front-area-loading-density seen by the particles as they circulate across the device.

For purposes of assessing energy broadening (upscattering and downscattering) of the convergent ion flow, the scatterings that are effective are only those in which the scattered ions have significant energy themselves. Scatterings that occur among low energy ions at the periphery of the system (or at or near the ion injection point within the system) will not lead to energy broadening (but may have other effects; these are treated in another EMC2 technical note).

> *Outer region ion-ion collisions may result in increased losses by deflection scattering, and reduced transverse momentum by isotropization of outer peripheral low-energy ions.*

Thus it is sufficient, in estimating $<nv>$ to take the ion speed as approximately constant at a mean core speed v_m in the (extensive) region of the bottom of the potential well, about the system center. This allows the product $<nv>$ to be written as $<nv> \approx <n>v_m$. Over this central region, the average ion density is comprised of two terms, that due to core ions within $r \le r_c$ and that due to ions outside the core convergence-limited radius $r = r_c$.

In the core region the ion density is assumed constant at n_c within the radius r_c. Outside r_c the results of simple analyses and extensive KXL/EKXL computations show that the density varies as the inverse square of the radial distance from the core, out to a radius $r = r_1$ at which the potential begins to rise towards the outer edge of the system. This point is found to be generally in the range $0.3 < <r_1> < 0.7$, and thus approaches that of the ion injection origin. The density variation of importance here can then be written simply as $n(r) = n_c(r_c/r)^2$. Using these the path-averaged ion density becomes

$$\langle n\rangle = 2\left(\frac{r_c}{R}\right)n_c \quad (3)$$

With this, and writing $v_m = (v_m/v_c)(2E/m_i)^{0.5}$, where v_c is the ion speed at the core at energy E, equation (2) can be solved for t_c as

$$t_c = \left[\frac{4f^2E^{1.5}}{\pi^2(Ze)^4 n_c}\right]\frac{R}{r_c}\frac{v_c}{v_m}\sqrt{\frac{m_i}{2}} \quad (4)$$

This equation gives the time for $\delta E/E = 2f$ broadening of ion energy for collisions at and around an energy E. But the ion losses of concern are due to upscattering (rather than downscattering), thus the effect of reduced collision cross-sections in higher energy collisions must also be taken into account in its estimation. As shown by

Rosenbluth, et al,[4,5] and noted previously,[6] the time required to upscatter to the maximum velocity $v_f = v_c(1+f)$ is increased by a factor of about one-half of the cube of the ratio of this velocity to the initial (mean) ion velocity v_c; $(1/2)(v_f/v_c)^3$. Since this is true for upscattering to all velocities between these two, the upscattering time factor averaged over all of the particles is just this cubic dependence weighted by the Maxwellian distribution, integrated over $v_c \le v \le v_r$. This gives a weighted average factor of approximately $(1/2)(1+f)^2$, which must be included in the time estimate of equation (4). This gives

$$t_c = \left[\frac{2(1+f)^2 f^2 E^{1.5}(v_e/v_m)}{\pi^2 (Ze)^4 n_c \langle r_c \rangle}\right]\sqrt{\frac{Am_p}{2}} \quad (5)$$

where $\langle r_c \rangle = (r_c/R)$ is the convergence ratio of the system,[7] and the ion mass is taken as $m_i = Am_p$ where A is ion mass number and m_p is the proton mass. Taking units of eV for E and all others in cgs gives

$$t_c = 7.1x10^6 \frac{(1+f)^2 f^2 E^{1.5}(v_c/v_m)\sqrt{A}}{n_c \langle r_c \rangle Z^4} \quad (6)$$

For the example case given earlier by Krall and Rosenberg,[8] with $Z = 1$, $A = 2$, $\langle r_c \rangle = 2 \times 10^{-2}$, $v_c/v_m \approx 1$, $f = 0.1$, $E = 1 \times 10^4$ eV, and $n_c = 1 \times 10^{12}/\text{cm}^3$, this equation gives $t_c = 6.1$ sec. Two other cases they examined later were for $\langle r_c \rangle = 1 \times 10^{-2}$, and $f = 0.5$, with other parameters as above, and for this same case but with higher density, $n_c = 1 \times 10^{18}/\text{cm}^3$, and energy, $E = 1 \times 10^5$ eV. For these equation (6) gives the upscattering times as $t_c = 565$ sec and $t_c = 1.79 \times 10^{-2}$ sec, respectively. All of these calculated values agree closely with results of their Fokker-Planck analysis for these examples. This simple formula thus can be used to model collision times for upscattering.

3. Collisional Losses

Such core collisionally-driven upscatter of ions will lead to expansion of their orbital motion to regions of higher magnetic field beyond that (outside) of their injection point. If the energy upscatter is great enough, the ions can leave the well entirely. This is the case considered here.

If upscatter is to lesser energies, circulation to larger radii may result in greater transverse energy/momentum generation by gyro motion within the surface field regions which, in turn, can result in an increase in the ion convergence ratio at the core, and thus yield lower core densities and poorer performance. This latter case is considered in another EMC2 technical note.

For this case of upscatter to direct escape energies, the collision time t_c (equations 5, 6) can be used to formulate a loss term in the algorithm for ion edge density (n_{ei} at $r = R$) time-dependence used in the EKXL code. Using the forms derived by King and Bussard,[9] and carrying out the algebra, gives

$$\frac{dn_{ei}}{dt} = \frac{1}{t_{trans}}\left\{\left[\frac{3n_{in}F_{trap}}{F_n F_t}\right] - n_{ei}\left[\frac{1}{G_i} + \frac{1}{F_u}\right]\right\} \quad (7)$$

Here, as before,[9] $F_t = (R/v_{in})/t_{trans}$, $F_n = \langle n_i \rangle / n_{ei}$ where the average ion density is $\langle n_i \rangle = N_i/(4\pi/3)R^3$, and F_{trap} is the anode height control function that limits the time-incremental density change used in the EKXL computation, and $\underline{F_u} = t_c/t_{trans}$. The term G_i is the ion current recirculation ratio in the system, as set by ion loss mechanisms other than direct escape by collisional upscattering (e.g. by particle cusp losses). Implementation of this algorithm has been made in the EKXL code, version 2.0. Calculation runs made with this code show that ion-ion core upscattering collisions have virtually no effect in systems of interest at central densities below about $1 \times 10^{18}/\text{cm}^3$. Since this density regime is already high enough to give interesting fusion output and system gain relative to electric drive power, the phenomenological effect of this model of upscattering is not dominant for most systems of practical interest.

Publishing History

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