Bremsstrahlung Radiation Losses in Polywell Systems

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The degree to which bremsstrahlung radiation constrains or limits system performance depends on the energy E_e of the electrons which are the principal source of this radiation, through their collisions with in-situ ions. This is true in those cases in which the electron energy is such that the electron speed exceeds the ion speed, at which condition the ions can be regarded as stationary targets for the electrons. If the electron energy is so low that the electron speed is comparable to the ion speed, then the ion energy must also be taken into account in computation of bremsstrahlung.

This latter condition can obtain only in the core of the Polywell device, and then only if the central virtual anode is nearly zero in height, and if the electron/ion collision rates are sufficiently small that no significant collisional heating of electrons can take place during the electron lifetime in the machine. Conditions under which these effects can be achieved in the Polywell system (in marked contrast to magnetic confinement LTE systems; in which they can NOT be achieved) are discussed further, following.

The basic expression for bremsstrahlung power density[1] in a mixture of electrons and j classes of ions, each with charge Z_j,

$$q_{br} = 1.69x10^{-32} E_e^{0.5} n_e \sum \left(n_j Z_j^2 \right) \quad (1)$$

shows this quite clearly. Here E_e is in eV and n in 1/cm³ for q_{br} in w/cm³.

In considering the range of fusion fuels it is important to note that the effect of $Z > 1$ can become quite profound on bremsstrahlung output, event at small Z. This is because, in the Polywell system, the charge density is very nearly neutral in the regions of highest density, where the principal radiation is generally, and

$$n_e = n_1 Z_1 + n_2 Z_2 \quad (1b)$$

while $$\sum \left(n_j Z_j^2 \right) = \left(n_1 Z_1^2 + n_2 Z_2^2 \right) \quad (1c)$$

where n_1 and n_2 are the local densities of the two fusion fuel species.

Writing these as fractions of the total ion density n_i, $n_1 = f_1 n_i$, $n_2 = f_2 n_i$, gives the bremsstrahlung power density as

$$q_{br} = 1.69x10^{-32} E_e^{0.5} n_i^2 \left(f_1 Z_1 + f_2 Z_2 \right) \left(f_1 Z_1^2 + f_2 Z_2^2 \right) \quad (2)$$

Now, the total bremsstrahlung power output is just this expression integrated over the total ion density and electron density distribution in the system volume. Thus

$$P_{br} = \int q_{br} 4\pi r dr; \; 0 \le r \le R \quad (2b)$$

It is readily shown[2] that this can be written in a simpler form as the integration over the convergence core volume, multiplied by a factor K_b which is the ratio of total bremsstrahlung power to core region power, thus

$$K_b = \frac{P_{br}(total)}{P_{br}(core)} \quad (2c)$$

Assuming that the fuel mixture remains constant throughout the region of the machine that is effective for generation of bremsstrahlung, this factor can be found by integration of equation (2) with the appropriate distributions.

> *The fuel mixture will NOT be uniform over the total volume in mixtures of high-Z and low-Z fuels; high-Z fuels will be excluded from the outer regions of the machine. However, essentially all of the bremsstrahlung comes from the near core region, where the fuel mixtures will be constant.*

In the region $0 \le r \le r_c$, both the ion density and the electron energy may be taken as constant, thus $n_i(r) = n_c$, and $E_e(r) = \eta_e E_o$, where E_o is the electron injection energy (and maximum possible well depth). The parameter $\eta_e = E_e(core)/E_o$ is the fractional energy of the core electrons, expressed as a virtual anode height parameter.

Note that $\eta_e \geq \eta$, where η is the height of the central virtual anode.[3]

From $r_c < r < r_k$ the density follows $n_i(r) = n_c(r_c/r)^2$ and the electron energy can be taken[3] as varying as $E_e(r) = \eta_e E_0(r_c/r)^2 + E_0(r/R)^3$, for a potential well with $m = 3$.

> *The actual well in an m = 3 system follows the "rollover" formula $\langle r\rangle^3 f_0(r)$, where $f_0(r) = 2/(1+\langle r\rangle^5)$. However, most of the bremsstrahlung comes from the inner regions where r << 1, in which the $\langle r\rangle^3$ approximation is quite good.*

This gives a reasonably good fit to the local potential and thus to the local electron energy, which is assumed to be in equilibrium with the potential. It can be shown[4] that the ion density increases in the region from $r_k < r < R$ from its value of n_k at r_k to $n_R = 3.0\ n_k$. For convenience this can be written as

$$n_i(r) = n_k\left(\frac{r}{r_k}\right)^q \quad (2d)$$

where the exponent is given by

$$q = \frac{3.0}{\ln\left(\frac{R}{r_k}\right)} \quad (2e)$$

Using these forms and integrating it is found that the first and second terms (i.e. in the region $r < r_k$) are dominant, and that the bremsstrahlung is split about 40% from the core and 60% from the central region immediately outside the core within the intermediate and inner mantle region, $r << r_k$. This result agrees with previous analyses[5] of the distribution. Thus $K_b = 2.50$ and the bremsstrahlung power is given by

$$P_{br} = 1.69x10^{-32}\left(f_1 Z_1 + f_2 Z_2\right)\left(f_1 Z_1^2 + f_2 Z_2^2\right) K_b n_c^2 E_e^{0.5} \quad (3)$$

in watts for E_e in eV, where n_c is the ion (and electron) density in the core.

The total fusion power in the system can be written in terms of the local fusion power density

$$q_f = b_{ij}\left(n_i(r)\right)^2 \sigma_f(E) v_i(E) E_f \quad (4)$$

integrated over the system. Here b_{ij} accounts for the possible number of interactions among differing and like specie fuels. In like fuels (e.g. DD) $b_{ij} = 0.5$, and for unlike fuels (e.g. DT)

$$b_{ij} = f_1 f_2 = f_1\left(1-f_1\right) = f_2\left(1-f_2\right) \quad (4b)$$

The maximum value of b_{ij} for unlike fuels requires that $f_1 = f_2 = 0.5$ for which $b_{ij} = 0.25$.

In a similar fashion to the bremsstrahlung analysis, above, the total fusion power can be expressed in terms of that generated within the core and that outside, by the ratio

$$K_f = \frac{P_f(total)}{P_f(core)} \quad (4c)$$

However, here the ion energy distribution differs from that for the electrons, as the system is nowhere in LTE and the ions are "cold" where the electrons are "hot", and vice versa. The ion energy varies as

$$\begin{aligned} E_i(r) &= (1-\eta)E_0\left(\frac{r_c}{r}\right)^2 + E_0\left(\frac{r}{R}\right)^3 \\ &= E_0\left(1-\langle r\rangle^3\right) - \eta E_0\left(\frac{r_c}{r}\right)^2 \end{aligned} \quad (4d)$$

Detailed calculations of fusion power density distribution and total fusion power output have been made for a variety of systems, using the EKXL v.4.1 code. Results[6,7] of these show that both the power density distribution and total power are functions of the central virtual anode height (and thus of the allowed ion current). The variation is such that, as the anode height increases, less of the fusion power is generated within the core convergence radius $\langle r_c\rangle$, and more comes from the region immediately outside ($r < 10\ r_c$) this core. As the anode height factor (nu) approaches unity, the core-generated power drops to zero and all of the power comes from outside the core. From this work it is found that the factor K_f varies as

$$K_f = \frac{2\left(1-\eta_0\right)}{\left(1-\eta\right)} \quad (5)$$

where η_0 is that value at which the in-core and out-of-core contributions are equal. Typically, $\eta_0 \approx 0.167$.

With this and noting the ion collisional speed in the CM system as given by

$$v_i = \sqrt{\frac{2E_i}{m_{pi}}} \quad (5b)$$

where m_p is proton mass and

$$M_i = \frac{m_1 m_2}{\left(m_1+m_2\right)m_p} \quad (5c)$$

is the normalized reduced mass of the ions, the total fusion power can be written as

$$P_f = 0.1 b_{ij} K_f n_c^2 \left[\sigma_f \left(E_c \right) \right] \left(\frac{2E_c}{m_p M_i} \right)^{0.5} E_f k_e^{1.5} \quad (6)$$

in watts, for k_e = 1.6x10[-12] ergs/eV, the fusion reaction energy E_f in MeV, and the core ion energy E_c in eV. Net power output requires that the ratio $P_{fb} = P_f / P_b$ be greater than unity. From equations(4) and (6) this becomes

$$P_{fb} = \frac{K_f b_{ij} \left(\frac{2}{m_p M_i} \right)^{0.5} \left(\sigma_f E_f \right) k_e^{1.5} E_c^{0.5}}{K_b \left[F_2(Z) \right] 1.69x10^{-31} E_e^{0.5}} \quad (7)$$

where

$$F_2(Z) = \left(f_1 Z_1 + f_2 Z_2 \right) \left(f_1 Z_1^2 + f_2 Z_2^2 \right) \quad (7b)$$

Specializing to the case where one fuel is singly-charged ($Z_1 = 1$) and noting that $f_1 + f_2 = 1$, gives

$$F_2(Z) = \left[1 + \left(Z_2 - 1 \right) f_2 \right] \left[1 + f_2 \left(Z_2^2 - 1 \right) \right] \quad (7c)$$

With this and writing $E_c = (1-\eta) E_o$, $E_e = n_e E_o$ equation (7) becomes

$$P_{fb} = \frac{K_f \left[F_3(Z) \right] \left(\frac{2}{m_p M_i} \right)^{0.5} \left(\sigma_f E_f \right) k_e^{1.5}}{k_b 1.69x10^{-31} \left[\frac{\eta_e}{(1-\eta)} \right]^{0.5}} \quad (8)$$

Here the function

$$F_3(Z) = \frac{b_{ij}}{F_2(Z)} = \frac{\left(1 - f_2\right) f_2}{F_2(Z)} \quad (8b)$$

Evidently there is an optimum value of the high-Z fuel fraction f_2, that will give a maximum fusion-to-bremsstrahlung ratio. This is found by differentiation of $F_3(Z)$ to be

$$f_{2'opt} = \frac{1}{\left(Z_2^{1.5} + 1 \right)} \quad (9)$$

and the optimum ratio of $Z_1 = 1$ to high-Z fuels is

$$f_{12} = \frac{f_1}{f_2} = Z_2^{1.5} \quad (9b)$$

Thus, for D^3He, $f_2 = f_{He} = 0.261$, $f_{12} = 2.83$, while for $p^{11}B$, $f_2 = f_B = 0.082$, and $f_{12} = 11.2$. In the D^3He case the system must be rich in D, which leads to a larger fraction of DD reactions and thus to higher neutron radiation output than for 50:50 or lesser mixtures. The $p^{11}B$ case is very proton-rich, which leads to much smaller power output from a given size of device which will, in turn, drive the system to larger sizes and higher *B* fields.

The maximum value of P_{fb} is thus determined by the natural properties of the fuels, the fusion cross-section (thus by the injection energy and well depth) and the energy of electrons in the central core. Using equations(5) and (8) and taking operation at optimum conditions (equation 9), the ratio P_{fb} can be written as

$$P_{fb} = F_b \left(f, Z, M_i, E_f \right) \sigma_{fb} \left(E_c \right) \frac{2\left(1 - \eta_0\right)}{K_b} \left[\frac{1}{\eta (1-\eta)} \right]^{0.5} \quad (10)$$

for σ_{fb} in barns (b), taken at core ion energy, $E_c = (1-\eta) E_o$, and the core electron energy factor has been set at $\eta_e = \eta$. The inherent values for the functional term, F_b are given in Table 1, below, for optimum mixtures and for 50:50: (equals) mixtures of each of the fuels shown. Also shown is a very He-rich D^3He case, to approximate "radiation-free" (i.e. insignificant DD reactions) operation such that NO shielding is required with this fuel combination.

Table 1 — Fusion to Bremsstrahlung Factors for Various Fuels

	Optimum Fuel Mixtures				50:50 Mix		1:1000
Fuel	DT	DD	D^3He	$p^{11}B$	D^3He	$p^{11}B$	D^3He
E_f (MeV)	17.6	3.65	18.3	8.7	18.3	8.7	18.3
M_i	1.2	1.00	1.20	0.92	1.20	0.92	1.20
f_{2opt}	0.50	—	0.26	0.082	0.5	0.5	0.999
F_b	57.7	23.9	18.8	2.28	13.0	0.76	0.22

Note that the energy per fusion event is lower for DD than is frequently quoted[8,9] for complete burning of all the products of the initial DD reaction. This is because the fusion products always escape the core of the electrostatic system and are not used directly in the burn cycle within the confined core region. Also note that the F factor for D^3He drops drastically as the mixture ratio is changed to seek nearly-neutron-free fusion power generation, so that D^3He systems than(sic, that) can be operated without significant radiation shielding have F values less than those for $p^{11}B$, which has no direct neutron output.

Since $K_b = 2.5$, $\eta_o = 0.167$, and η must be small for effective operation, equation (10) can be approximated as

$$P_{fb} = 0.667 \frac{F_b \sigma_{fb}}{\eta^{0.5}} \quad \text{(11a)}$$

For fusion power generation to exceed bremsstrahlung then requires that $P_{fb} > 1$, which will occur only when

$$\sigma_{fb} > \frac{K_b \sqrt{\eta(1-\eta)}}{2F_b(1-\eta_0)} = \frac{1.34\sqrt{\eta}}{F_b} \quad \text{(11c)}$$

as a necessary criterion for net fusion power.

Determination of the minimum possible value of η_e that can be achieved is of some complexity, for it involves consideration of ion/electron up- and down-scattering collisions in different regions of the system, as well as of the ratio of ion current to electron drive current used to establish and maintain the potential well.

The EXKL code runs have shown that minimum virtual anode heights will always be at or above $(1\text{-}a_q) \approx 0.005$ (i.e. the maximum well depth never gets closer to injection energy than $a_q = e\Phi_{max}/E_o \approx 0.995$). If the electrons could be kept at this energy, then $\eta_e = 0.005$, and bremsstrahlung losses will always be much less than fusion power generation capabilities The question here is the degree to which ion/electron collisions in the core region can transfer ion energy to the electrons sufficient to raise η_e significantly above this level. It is thus necessary to examine the ion/electron collisional energy exchange process in some detail.

Such energy exchange, will, of course, occur in the core, mantle and edge regions. In the outer mantle, beyond the electron "stagnation" radius $\langle r_f \rangle = (dE_{o\perp}/E_o)^{0.5}$ and in the edge region, electron/ion collisions will "cool" the electrons, while in the region inside $\langle r_f \rangle$ ions will "heat" electrons. The important feature is the balance between up- and down-scattering in a single pass of an electron through the system. If the up-scattering in core region passage is removed by the down-scattering in extra-$\langle r_f \rangle$ collisions with cold ions, then the core electron energy will be stable. This stable electron energy is thus a result of competing collisional processes in the spatially-alternating non-LTE ion/electron distribution of the system. Analysis of these processes shows the stable up-scattered electron core energy (at which equality of ion/electron energy exchange will take place in the "heating" and "cooling" sections of the system) to be approximately

$$\eta_e = \left(\frac{N_{core}}{N_{tot}}\right) \frac{\langle r_f \rangle^4}{10\langle r_c \rangle^{0.5}} \quad \text{(11b)}$$

Here N_{core}/N_{tot} is the fraction of electrons that "see" the core, equivalent to the single-pass core-sampling frequency of electrons circulating in the system, and the factor of 10x comes from analysis of the edge/mantle cooling collisions. The sampling frequency can be estimated by a simple ratio of electron number in each region. Using the density distributions cited above and integrating over the complete system gives this approximately as $\langle r_c \rangle/3$. Substituting into equation (11) yields

$$\eta_e \approx \frac{\langle r_f \rangle^4 \langle r_c \rangle^{0.5}}{30} \quad \text{(12)}$$

If $<r_f> = 0.707$, for example (a highly-spread electron distribution), and the convergence radius is take to be $<r_c> = 10^{-2}$, then $\eta_e = 0.83 \times 10^{-3}$ is found. If $<r_f> = 1.0$ (the maximum possible value), then $\eta_e = 3.3 \times 10^{-3}$, above the ion-driven virtual anode height. Taking this as $\eta = 5 \times 10^{-3}$, as discussed above (for maximum a_q), yields an electron core energy of $\eta_e = 5.8 \times 10^{-3}$ to 8.3×10^{-3} as an absolute minimum for a system constrained to operate at the lowest possible virtual anode height.

It is obvious, from equations (10,11), that maximum P_{fb} will be found for the highest possible value of the fusion cross-section, σ_{fb}, thus for operation at that energy at the peak of cross-section variation. However, this peak energy is not necessarily optimum for the competition of fusion with synchrotron radiation power, P_{sy}. A study of the synchrotron question has shown[10] that the optimum well depths (injection energies) for maximum $P_{fs} = P_f/P_{sy}$ are as listed in Table 2, below.

Table 2 — Optimum Operation for Fusion/Bremsstrahlung Power Balance

	Optimum Fuel Mixtures				50:50 Mix		1:1000
Fuel	DT	DD	D^3He	$p^{11}B$	D^3He	$p^{11}B$	D^3He
E_f (MeV)	17.6	3.65	18.3	8.7	18.3	8.7	18.3
f_{2opt}	0.50	—	0.26	0.082	0.5	0.5	0.999
F_b	57.7	23.9	18.8	2.28	13.0	0.76	0.22

Fusion-to-bremsstrahlung power ratios, P_{fb}, are given in Table 2 for each of the fuel mixtures used in Table 1, above, for a variety of well depth or injection energy conditions. These calculations have been made using a practical minimum value of $\eta_e = 0.01$ for the core electron energy ratio; from equation (11) this gives $P_{fb} = 6.667 F_b \sigma_{fb}$.

Note from Table 2 that all of the fuels can operate at bremsstrahlung-optimum mixture ratios with negligible bremsstrahlung losses if the electron energy state can be kept as low as assumed above. However, losses with $p^{11}B$ at 50:50 mixtures are significant in comparison with fusion power generation, and losses in D^3He at the 1:1000 mixture ratio taken for radiation-free operation are prohibitive. DT is able to operate easily at all conditions, and can function quite well at any virtual anode height condition. In fact, all of the fuels at optimum mixture conditions can operate with minimal bremsstrahlung at anode heights of $\eta_e \leq 0.15$, or so. Also, it is clear that DD and D^3He offer similar performance envelopes at optimum mixture conditions while D^3He is similar to $p^{11}B$ when operated in a non-radiative mode.

Finally, note that bremsstrahlung is a more pervasive constraint than synchrotron radiation because the latter can be reflected by bounding metal walls, and the loss fractions are much less than given above when the effects of resonance self-absorption within the plasma are taken into account.[10] Bremsstrahlung power generation is inherent in the plasma mixture, it can not be suppressed, reflected or self-absorbed — it is simply a loss mechanism.

In conclusion it is gratifying to see that all four of the fuel combinations can be made to work effectively in the Polywell system; a result that is not true for use of these fuel combinations in "conventional" magnetic, Maxwellian fusion systems in local thermodynamic equilibrium.

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