Collisional Equilibration

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1. Ion/Ion Collisions and Maxwellian Equilibrium

Consider two-body collisions among monoenergetic particles of density n_c (1/cm³), charge Z, at energy E_c (eV). The collision frequency f_2 is taken as the inverse of the two-body energy exchange (90° deflection) or slowing-down time, t_s (Delcroix),[1] given by

$$f_2 = \frac{1}{t_s} = 1.35x10^{-7}\frac{n_c Z^4 LN(\Gamma)}{\sqrt{A}E_c^{1.5}} \quad (1)$$

Here A is the ion mass number and Γ is the Coulomb impact parameter. The two-body collision rate density is just $q_2 = n_c f_2$. Since fusion density is

$$q_f = b_{ij} n_c^2 \sigma_f v_c \quad (2)$$

where b_{ij} is 0.25 (0.5) for differing (identical) ions, the ratio of two-body to fusion collisions is $q_{2f} = q_2/q_f$

$$q_{2f} = 1.08x10^{11}\frac{Z^4 LN(\Gamma)}{b_{ij}\sigma_f E_c^2} \quad (3)$$

for collision energy (CM frame) E_c in eV and σ_f in barns. Here the particle collision speed has been taken to be $v_c = [2E_c/mA]^{0.5}$ for monoenergetic particles (all fusion reactions have been assumed to be at 90° incidence) and σ_f is to be evaluated at v_c.

If $(\sigma_f v_c)$ is integrated over the actual collisional energy distribution from the initial radially-monoenergetic ion energy distribution, fusions are more probable and the coefficient in equation (3) will become considerably smaller than given above. For example, for DT, if E_c is taken as 10 keV, and the initial collision energy distribution $E_c(\theta) = E_c(1\text{-}\cos\theta)$ is used, then the integrated average value of $<\sigma v>$ is about 3.4 times larger than for the monoenergetic calculation. Thus, initially — at this mean energy — the ratio q_{2f} will be 0.29 times that calculated from equation (3); at E_c = 20 keV the factor is about 0.62.

The mean time for "approximate Maxwellianization" (t_{mx}) can be taken as the collisional slowing down time (t_s), from equation (1), at the core high density conditions, divided by the fractional time (g_i) that any given ion spends in this region, thus $t_{mx} = t_s/g_i$. However, as shown by Rosenbluth *et al*,[2] the time (t_L) to Maxwellianize particles by upscatter to velocity v_L in the high energy "tail" of the distribution is greater than this (because of cross-section decrease with energy) by a factor of about $(1/2)(v_L/v_c)^3$, thus the time to Maxwellianize particles to energy E_L in the "tail" is

$$t_L = \frac{t_{mx}}{2}\sqrt{\frac{E_L}{E_c}} \quad (3b)$$

It is evident that an increase from E_c to 6 E_c, say, requires about $7.2t_{mx}$. If E_c is initially at 10 keV, for example, then the tail of the distribution will be filled at 60 keV in this time. Now) at this higher energy the fusion cross-section for the fast particles (of DT) with bulk ions is about 20 times larger than for monoenergetic 10 keV ions alone) thus their fusion reaction rate will be higher by about 20(60/2)0.5 ≈ 110 than that of the (assumed) initial 10 keV population. These two effects plus the increase in ion speed with $(E_c)^{0.5}$ are responsible for the well-known fact that most fusion reactions in thermalized DT at ca. 10 keV come from regions at 50-60 keV in the Maxwellian tail.

> At t = t the mean energy distribution will have relaxed so that the low- and mid-range distribution is "approximately Maxwellian."

For monoenergetic ions at 10 keV, q_{2f} is about q2f ≈ 3.1 x 10^4, while at 60 keV, $q_{2f} \approx 210$) corrected for all-angle collisions. With such collisionality the bulk collisions in the central core ($r \le r_o$) will tend to distort the core in energy distribution from its initial form. However, it is important to note that the two-body collisions in equations (1) and (3) *do not involve the same ions from one moment to the next.* This is because each ion resides in the core only a small fraction of its transit time across the system, thus the rate of collisional energy scattering is much less than would be adduced from the calculation of q_{2f}, above.

The time any given ion spends in the core is just $\delta t_c = r_o/v_c$) while the time required for a system transit is

$$\delta t_R = 2\int_0^R \frac{dr}{v(r)} \quad (3c)$$

For an assumed parabolic potential well shape the ion speed is given crudely by $v(r) = v_c[1-\langle r\rangle^2]^{0.5}$, where $\langle r\rangle = (r/R)$. With this, the transit time is estimated as $\delta t_R \approx \pi R/v_c$, so that the fraction of time spent in the core is given by $g_i \approx r_o/\pi R = \langle r_o\rangle/\pi$.

Thus the number of upscattering two-body collisions per ion per fusion reaction becomes $q_{2fsc} = q_{2f}\langle r_o\rangle/\pi$, so that the previous examples give only $q_{2fsc} \approx 101$ for $E_c = 1 \times 10^4$ eV and $q_{2fsc} = 0.67$ for $E_c = 6 \times 10^4$ eV. Since Maxwellianization requires at least t_s for ions in the bulk and $(t_s/2)(E_L/E_c)^{1.5}$ for the tail ions, it is evident that full Maxwellianization can not occur for well depths of 25-30 keV or greater, and that the high energy tail distribution will never be filled for such mean core energies.

Core mean energies below this collisional energy range can result in Maxwellian distributions well out into the tail. For Maxwellian distributions in the core) the q_{2f} ratio can be written for DT with a Maxwell-averaged $\langle\sigma v\rangle$ at core mean energy E_m as

$$q_{2mf} = 3.2x10^{-7}\frac{LN(\Gamma)}{\langle\sigma v\rangle E_m^{1.5}} \quad (4)$$

Taking the Coulomb factor to be $\mathrm{LN}(\Gamma) = 10$ as typical, this yields $q_{2mf} \approx 200$ at $E_m \approx 5 \times 10^4$ eV as the average number of two-body energy exchange collisions per average fusion event. However here, as before, ions in the tail can have much higher fusion reaction rates, thus will tend to be reacted before reaching full Maxwellian density equilibrium for $E_m <<$ E- at-peak-cross-section.

Now, ions with energies $E_L > E_m$ in the Maxwellian tail will circulate to radial positions further out than the ion injection radius, $r_{inj} < R$. As the Maxwellian tail is built up by continuous collisions within the small central core, these fast ions will climb further and further out until they are eventually lost to the system. If the well is three times deeper than the potential at the injection radius, fast ions must have resided in the core collision system for $(3^{1.5})/2 \approx 2.6$ times the two-body collision time, t_s. If the well is six times deeper, the fast ion residence time would be ca. $7.2t_s$. In this case the fast ions at $E_L > 6E_m$ will all escape from the system. The energy they carry at escape is small, however, in that their fast tail energy is lost to the confining well before their departure from the system.

Ion collisional makeup will be required to supply the thus-truncated tail, but at a rate of only about $2(E_m/E_L)^{1.5} \approx 0.13$ times that of the basic two-body collision process. This energy exchange rate is just $q_2 = nf_2$, from equation (1), and is $q_2 \approx 1.0 \times 10^{21}$ collisions/second/cm^3 in the core with $n_c = 1 \times 10^{17}$/cm^3 at $E_c = 5 \times 10^4$ eV. If the core radius is $r = 1$ cm, the total collision rate $Q_2 = q_2$(core volume) is $Q \approx 4.3 \times 10^{21}$ collisions/second. However, any given ion is only in the high density core region for a fraction g_i of its lifetime within the system, thus the rate of upscattering of each ion in the system will be less by this factor than would be estimated from the expression for loss to a sink at energy E_L above core energy E_c (or E_m).

Thus the actual loss rate for this example would be

$$Q_L = 2g_iQ_2\left(\frac{E_c}{E_L}\right)^{1.5} \quad (5)$$

The ion in-core time fraction is (as previously shown) about $g_i = \langle r_o\rangle/\pi$. With this, for a convergence ratio of $\langle r_o\rangle = 10^{-2}$) the previous example gives a DT mass loss rate of $(dM/dt)_{DT} \approx 2.74 \times 10^{-4}$ gm/sec to $E_L > 6E_c$. This is a volumetric loss rate of about 1.5×10^{-3} liter atmosphere/second, adding a leakage gas load to the vacuum pumping system. Note that the energy of these escaping ions outside the system is not $6E_m$ but is very small; most of their energy has been given back to the well, this loss should not create any significant problem of ion sputtering external to the system. If $E_c = 2 \times 10^4$ eV, the loss rate is four times greater for $E_L > 6\ E_m$ and eleven times greater for $E_L > 3\ E_m$. Loss rates for this latter case would be 1.87 mg/second or 1.68×10^{-2} liter atmosphere/second. If pumping capacity is 3×10^4 liter atmosphere/second this would limit the system base pressure to about 5.7×10^{-7} atmosphere.

The system net energy gain (balance) will be little affected, since most energy loss is due to electron escape through the cusps. Note that this gain *is* reduced by the factor of increased electron injection energy required to create a deeper well to hold upscattered ions versus the minimum well depth needed for core ion energy alone. But deeper-than core-energy wells are required anyway, to allow ion injection deep enough within the magnetic field surface to avoid excessive transverse ion momentum generation in core ion recirculating flow.

2. Electron Thermalization

Electron collisional thermalization will also occur, but principally in regions of large density and in these regions the electron energy is low. In the core the relevant collision type is that of ion collisional heating of electrons. The relaxation time for equilibration in such collisions is (m_i/m_e) longer than that for electron self-collisional relaxation. Thus, since ion/ion two-body collisional relaxation is slower by the square root of this factor, ion heating of electrons will be slower than ion two-body collisions by this same factor, $(m_i/m_e)^{0.5}$.

There are two points of interest here. First is the electron thermalization time compared to the electron lifetime in the high density core, as limited by G_j transits before cusp escape. Spitzer[3] gives this time as

$$t_{eq} = \left[\frac{4.0x10^6 A_e A_i}{n_i Z^2 LN(\Gamma)}\right]\left[\frac{E_e}{A_e} + \frac{E_i}{A_i}\right] \quad (6)$$

For a core radius of r = 1 cm, an assumed electron energy of e.g. 1 keV in the core, so that $v_{ec} \approx 1.8 \times 10^9$ cm/sec, and an electron current recirculation factor of $G_j = 1 \times 10^5$, the core lifetime is $t_{ec} \approx 1.1 \times 10^{-4}$ sec. At this electron energy, with ion energy of $E_c = 5 \times 10^4$ eV and LN(Γ) = 201 the ion/electron equilibration time for $n_{i,e} = 1.0 \times 10^{17}$/cm^3, for example, would be $t_{eq} \approx 6.7 \times 10^{-6}$ sec from equation (6), above. Thus, 1 keV electrons will be well thermalized after only a few thousand passes through the core. The assumed (low) electron energy therefore is not valid.

At the other extreme, assume that the electrons have energy of 5×10^4 eV (as though already in equilibrium with core ions). Then the equilibration time is $t_{eq} \approx 2.1 \times 10^{-3}$ sec, while the lifetime in the core is only $t_{ec} \approx 1.6 \times 10^{-5}$; about 130 times less than this.

Thus electrons in this example will not remain cold, nor will they reach ion temperatures/energies. Rather, the electron energy will tend to reach an equilibrium value when $t_{ec} = t_{eq}$. ,or when the core lifetime

$$\frac{2G_j r_0}{v_e} \approx 1.7x10^8 \frac{E_e^{1.5}}{n_i Z^2 LN(\Gamma)} \quad (6b)$$

For $v_e = (2E_e/m_e)^{0.5}$ this gives the equilibrium core electron energy as

$$E_{eleq} \approx 1.45x10^{-8} Z\sqrt{n_i r_0 G_j LN(\Gamma)} \text{ (eV)} \quad (6c)$$

If $n_i = 1 \times 10^{17}$/cm^3, r = 1 cm, LN(Γ) = 20, then $E_{eleq} = 6.7 \times 10^3$ eV. For higher ion density, e.g. $n_i = 1 \times 10^{18}$/cm^3) this is still only ≈20 keV. However if $G_j \rightarrow 1 \times 10^6$ the calculated equilibrium electron temperature for this last case becomes an unphysical 63 keV, and electron losses would become greatly enhanced by core thermalization.

For more realistic, lesser (initial experimental) conditions) electron thermalization is not significant. For example, for $G_j = 1 \times 10^3$, $n_i = 1 \times 10^{12}$/cm^3, and r = 1 cm) this simple model gives $E_{eleq} \approx 2.1$ eV. Since this is very much less than $(m_e/m_i)E_i$ for all $E_i > 2 \times 10^4$ eV, the ion collision rate with electrons would dominate, and the equilibration electron temperature due to ion/electron collisions is expected to be only $E_{eleq} \approx (m_e/m_i)E_i$. This does not take into account electron acceleration by a central virtual anode) which can raise the electron energy. In general it is expected that core collisional equilibration of electrons will be insignificant at planned laboratory experimental conditions and will become noticeable but not dominant as reactor conditions are approached.

Electron collisions will also occur outside the core region. Here the electron density will tend to follow the ion density. Taking a parabolic distribution as illustrative,

$$n_e(r) = n_{e0}\left(\frac{R}{r}\right)^2 \quad (6d)$$

as typical, where n_{eo} is electron surface density

$$n_{e0} = G_j n_{elinj} \quad (6e)$$

and the electron energy variation also as parabolic,

$$E_e(r) = E_{eo}\left(\frac{r}{R}\right)^2 \quad (6f)$$

yields the functional dependence of the equilibration time as

$$t_{eq} \approx \frac{KE_{e0}^{1.5}\langle r\rangle^5}{n_{e0}}\left(\frac{r}{r_0}\right)^5 \quad (6g)$$

where K is a factor containing design parameters and constants of the system. From this it is evident that electron equilibration will proceed much more slowly as $r \rightarrow R$ than as $r \rightarrow r_o$, and hence can be neglected at least to first order.

3. Ion/Ion Collisional Dispersion

Returning to ion collisions, it is of interest to estimate spreading of initially-radially-monoenergetic energy distributions and of the ion spatial convergence radius due to ion/ion self-collisions in the core. Lovberg[4] has shown that these effects can be kept non-dominant in system operation.

The first complete analysis of collisional equilibration of particles with initial uniform and equal energy was carried out by Maxwell[5] in 1859, who showed that the stable state distribution of energy (or velocity) was (what we now call) the Maxwell (or Maxwell-Boltzmann) distribution

$$f_M = \left(\frac{m}{2\pi kT}\right)^{1.5} EXP\left(\frac{-mv^2}{2kT}\right) \quad (7)$$

In the central core, ion/ion collisions will produce this distribution, albeit with a truncated tail (for some conditions, as explained above) with the consequences described previously. This distribution is achieved by the combined effect of a very large number of small-angle collisions of each particle with the Coulomb fields of the surrounding particles that can affect its motion. These are limited to those contained within a sphere around each particle with radius equal to the Debye shielding radius

$$L_D = \left(\frac{kT}{4\pi ne^2} \right)^{0.5} \quad (8)$$

For ion core energies $E_c > 1 \times 10^3$ eV, the Debye sphere will contain a very large number of ions for any density $n < 1 \times 10^{19}/\text{cm}^3$, thus most collisions-deflections will be due to distant small-angle encounters. Anyone collision will yield only a small deflection and momentum transfer to the deflected particle. The next collision will be uncorrelated in direction, and successive collisions equally so. The net result is that the spread of particle motion and momentum (and thus of energy) will proceed as a collisional random-walk process until momentum equivalent to the particle initial momentum is exchanged with the plasma system.

At this point the particle will have been deflected approximately $\pi/2$ (90°) from its initial course and can be viewed as then having undergone an "energy exchange collision" of the sort used in the preceding sections herewith. While the time to reach the state of this "collision" and the energy exchange associated with it are both useful in zero order analyses (as above) they do not disclose details of the random walk process that are important in assessing first order effects in core collisionality.

To illustrate these it is useful to consider the random walk problem. Following Jeans,[6] the random collisional one-dimensional motion of a particle traversing a mean free path £, between collisions is describable by the probability P that it will have moved a distance s£, in N collisions by

$$P = \frac{N!}{p!q!2^N} \quad (9)$$

where $p+q = N$ and $p-q = s$. Noting that N is very large in problems of interest and that s is small relative to N allows this factorial representation to be reduced by Stirling's approximation (for $n!$ with n large) to the familiar Gaussian form

$$P(s) = \sqrt{\frac{2}{\pi N}} EXP\left(\frac{-s^2}{2N} \right) \quad (10)$$

This is the probability that a particle has moved s mean free paths in N collisions in one-dimensional motion. Rewriting this in terms of particle speed v and time t) and extending to three dimensions, yields

$$P(r) = \left(\frac{2£}{\pi vt} \right)^{1.5} EXP\left(\frac{-r^2}{2vt£} \right) \quad (11)$$

as the probability of diffusion to radius r from the origin, in time t, expressed in one-dimensional spherical coordinates.

This can be applied in two ways to the present problem of diffusion due to numerous small angle collisions. One way is to determine the radial displacement associated with a given number of collisions, and thus to obtain a measure of ion dispersion and associated core convergence defocus sing due to same. Another is to estimate the energy change due to small angle collisions over a given time or collision number.

This latter is straightforward, simply noting that the probability P equally defines diffusion in energy space, where $r \rightarrow E$ (E is total energy spread), with £ $\rightarrow \delta E_o$ as the energy change per collision (the "energy mean free path"), and $v \rightarrow f\delta E_o$ as the "speed" in energy space, thus

$$P(E) = \left[\frac{2}{\pi ft} \right]^{1.5} EXP\left(\frac{-E^2}{2ft(\delta E_0)^2} \right) \quad (12)$$

Here the energy exchange per collision is just $\delta E_o = \pi(Ze)^2/L_D$, where L_D is the Debye length, $L_D = (kT/4\pi ne^2)^{0.5}$, as usual, or $L_D = (E_c/6\pi ne^2)^{0.5}$ for the thermalized case where $E_c = (3/2)kT$.

The mean free path in coordinate space is just £ $= 1/n\sigma_D$ where σ_D is the Debye sphere small angle collision cross-section (ff.. Lovberg),[4] $\sigma_D = \pi(L_D)^2$. The collision frequency is then simply $f = v/£ = (nv\sigma_D)$, where $v = (2E_c/m_i)^{0.5}$. Now the probability that a particle will scatter into *some* energy $0 < E < \infty$ must be exactly unity. Thus the energy probability P(E) can be normalized to

$$C\int_0^\infty P(E)dE = 1 \quad (12b)$$

from which it is found that the coefficient $C = (mft/2\delta E_o)$, so that the probability of scattering to an energy displacement E, from initial energy, per unit energy) is just

$$P_0(E) = \sqrt{\frac{2}{\pi ft}} \frac{1}{\delta E_0} EXP\left(\frac{-E^2}{2ft(\delta E_0)^2} \right) \quad (13)$$

With this the most probable value of the scattered energy $\langle E(t) \rangle$ at any time) from small-angle collisions about an initial energy state (i.e. the Gaussian full-width at half-maximum amplitude) is found from

$$\langle E(t) \rangle = \int_0^\infty EP_0(E)dE = \sqrt{\frac{2ft}{\pi}} \delta E_0 \quad (14)$$

which is analogous to Lovberg's equation (12) and its predecessor. Using the Debye collision cross-section as before, this gives an energy dispersion due to small-angle scattering over time t of

$$\langle E(t)\rangle = \pi\sqrt{2nvt}\,(Ze)^2 \quad (15)$$

For 10 keV ions *uniformly* distributed throughout the system at a density of n = 1 x 10^{12}/cm³, and with a system lifetime of t = 1 x 10^{-2} sec (as in Lovberg's first example), this yields <E(t)> ≈ 632 eV or ±316 eV as the deviation from mean energy. Note that the collisional time assumed gives an integrated area density of (ion) collision centers along the ion path of about 1 x 10^{18}/cm² for any given ion transiting the uniform system.

As previously shown, the ion system transit time is about $\delta t_R = \pi(R/v_c)$, where v_c is ion speed at core conditions (assuming a relatively flat central region without significant virtual anode formation), Typically the transit time is $\delta t_R \approx 3.1 \times 10^{-6}$ sec for R = 100 cm and $E_c = 1 \times 10^4$ eV; this corresponds to about $G_i \approx 3.2 \times 10^3$ ion transits through the system. This is roughly equivalent to an electron current recirculation factor (outside the stagnation radius r_f) of $G_j \approx G_i(m_i/m_e)^{0.5} \approx 1.4 \times 10^5$.

The uniform distribution of ions assumed above is of course) not realistic for the system. Rather) the ion distribution will vary approximately as $(1/r^2)$ outside a core of radius r_o, and will increase faster within the core. With this distribution, and assuming a 3:1 density jump at the core boundary, as used for convenience in earlier calculations (e.g. DTI/1989,[7] Lovberg),[4] the integrated value of

$$\langle nvt\rangle = 2G_i\int_0^R n(r)dr \quad (15b)$$

is about $3.32(G_i n_c r_o)$. For the G_i value above, this gives a much smaller value for collisional density than before; here $\langle nvt\rangle \approx 1.1 \times 10^{16}$/cm2 which yields an energy spread of <E(t)> = 63 eV or a half-width of ±31 eV.

As a final example consider a core density of nc = 3 x 10^{17}/cm³, a core ion energy of $E_c = 3 \times 10^4$ eV, r_o = 1 cm and the same number of ion transits as in the previous case. Then, using the density distributions cited by Lovberg, the total ion path density per unit area is $\langle nvt\rangle = 3.92 \times 10^{21}$/cm², For this system, the energy spread becomes (equation 15) <E(t)> ≈ 2.82 x 10^4 eV = 28.2 keV, for a half-width of 14.1 keV. This spread seems acceptable over such a lifetime, even though the system is significantly (but not completely) Maxwellianized,

For such a quasi-Maxwellianized core-ion thermal energy (temperature) of E_c = 30 keV, the initial radial ion injection energy must be about $(3/2)E_c$ or 45 keV. The mean upscatter energy dispersion is, as above, about ±14.1 keV. Allowing for 2.5 times the mean ion temperature gives a well depth of 75 keV which is sufficient to trap the mean upscattered ions. Ions inserted into the well at the 45 keV point would be well inside the rippled *B* field region, and less prone to transverse momentum generation than for higher energy injection at larger radii. Lesser well depth could also be used, but at a price in ion loss rate, as previously described.

It is important to note, in this connection, that the upscatter energy dispersion given by equations (14, 15) and used above is only a mean value, and that the actual upscatter energy follows the Gaussian distribution of equation (12). This is another way of representing the eventual stable Maxwellian distribution that will obtain for long-lived trapping of core ions.

Note, also I that the fusion power output of the last example system will be

$$P_f = n^2\langle\sigma v\rangle E_f\frac{4\pi}{3}r_0^3F(\langle r_0\rangle) \quad (15c)$$

where

$$F(\langle r_0\rangle) = 1 + \frac{1-3\langle r_0\rangle}{3} \approx 1.32 \quad (15d)$$

For E_c = 30 keV, $\langle\sigma v\rangle \approx 5.7 \times 10^{-16}$ cm3/sec (Maxwellian average), the fusion power generation becomes $P_f \approx 8.0 \times 10^8$ watts or 800 Mw. Thus, operation with DT at conditions useful for fusion power production seem tractable within the constraints of collisional upscattering. Of course, well depths must be chosen large enough to avoid bulk two- and three-body collisional problems in all events, as these are both strongly well-depth dependent.

As a last topic, consider the *coordinate* spreading introduced by small angle scattering. The coordinate probability given by equation (11) can be normalized to unity, as was done for spreading in energy space, by integration over 0< r < ∞. Noting that the mean free path is £ = 1/$n\sigma_D$, carrying out the integration gives the normalizing coefficient as K = $(\pi vt)(n\sigma_D/2)^2$, and the probability of reaching a displacement r, per unit path length) becomes

$$P_0(r) = \sqrt{\frac{n\sigma_D}{2\pi vt}}EXP\left(-\frac{n\sigma_D r^2}{2vt}\right) \quad (16)$$

The most probable displacement over any given time period $\langle\delta r(t)\rangle$ is just the function $rP_o(r)$ integrated over the complete range of r. This yields

$$\langle\delta r(t)\rangle = \sqrt{\frac{vt}{2\pi n\sigma_D}} = \frac{\sqrt{6e^2t/\pi}}{(2mE_c)^{0.25}} \quad (17)$$

for $\sigma_D = \pi(L_D)^2$ and $L_D = (E_c/6\pi ne^2)^{0.5}$ as before. Using the previous examples) the mean displacement due to small angle collisional distortion of the initial energy distribution is found to be only $<\delta r(t)> = 3.8 \times 10^{-3}$ cm for $n_c = 1 \times 10^{12}$/cm3, $E_c = 1 \times 10^4$ eV, and $t = 1 \times 10^{-2}$ sec. For the more realistic case of non-uniform density, where $vt = 2G_iR \approx 6.2 \times 10^5$ cm (rather than 6.2×10^7 cm as in the initial case) this is found to be only 8.6×10^{-4} cm for the same ion recirculation factor G_i. It thus appears that energy exchange collisions will not contribute *directly* in any significant way to core convergence *spatial* broadening.

However, the upscattering of energy will cause ions to orbit to larger radii, and thus to enter regions of larger B field, in which the B field structure is more pronounced (as cf. to near-sphericity in the core region). This (outward) penetration of stronger and more rippled B fields will result in the transformation of a larger fraction of ion radial energy into transverse momentum than for orbits to smaller radii. This larger transverse momentum in turn can defocus the ions in their subsequent transits of the core region, and may even result in the trapping of ions in orbit paths well outside the original core radius.

The upscattering process thus results in core convergence broadening from the *indirect* effect of increased transverse momentum generation due to ion orbit expansion. In the absence of restoring focussing effects, this process can continue only until the ion core offset radius is at a distance approximately equal to half of the gyro radius of an ion calculated as with core energy in the cusp surface field) at which time the ion will cease to occupy a repetitive orbit, but will leave the system. Such restoring effects may be found in transverse collisions in the core[8] and/or at the outer radius of ion motion) as described by Rosenberg and Krall.[9] Further exploration of this coupling of energy upscattering to ion orbit distortion and momentum transformation is required to obtain a good understanding of these phenomena, and of their effects in various systems.

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