

Effective Gyro Hole Loss Radius and Diamagnetic Limit in Polywell Systems

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In the fully-filled model of a Polywell field system, the electron $\beta = 1$ surface at $\langle r_b \rangle = (r_b/R)$ is at or near the outer edge of the plasma. At this condition, the electrons are at maximum energy E_0 and speed and provide a pressure $p_e = 2/3 nE_0$ against the magnetic field B at that point and its magnetic pressure

$$p_b = \frac{B_b^2}{8\pi} \text{ (1b)}$$

This electron pressure is the averaged result of electron gyro motion in the external field envelope at various angles of incidence for the quasi-non-adiabatic region within the radius r_b .

At the position of the cusps, however, electrons approaching a cusp by gyro motion from shallow angle incidence around the cusp region will be able to escape through the cusp if their azimuthally-offset position of entrance into gyro motion in the surface field is within one-half gyro radius of the edge of a gyro radius hole around the cusp axis. Hence, the net effective loss radius for electrons through a cusp is approximately two gyro radii ($2r_{gy}$) at the surface field conditions, thus $k_L = 2$.

The foregoing heuristic argument applies strictly for a purely spherical surface field penetrated by gyro radius cusp "holes." In the actual case, interchange stability requirements will force the confining surface field at electron $\beta = 1$ to be not spherical but slightly convex inwards towards the center of the device. Thus the intercusp confining fields connect to the cusp hole fields not at right angles but at some shallower angle. This will result in an increase in the offset radius from the cusp axis from which surface gyro electrons can reach the

gyro radius cusp hole before returning to $r < r_b$. Because of this, the effective gyro hole radius ($r_L = k_L r_{gy}$) will be increased by a factor $k_L > 2$. Typically $2 < k_L < 3$ is expected for the system. This phenomenological result is supported by the approximate analysis of diamagnetic cusp behavior given, following.

Diamagnetic Cusp Limit

Now, electron gyromagnetic motion parallel to the external surface can (and will) produce internal currents that create magnetic fields of opposite sign to that of the externally-driven cusp field. Only those electrons whose gyro orbits encircle the cusp axis: have any net effect on this process. The volume of the diamagnetically-defective electrons is bounded roughly by a torus of major and minor radius equal to the gyro radius in the effective cusp field, thus the cross-sectional area A_c of the diamagnetic current flow is

$$A_c = \pi r_{gy}^2 \text{ (1)}$$

The maximum electron diamagnetic current (without collisions) is then

$$I_{de} = n_{e0} v_{e0} A_c = \frac{\pi r_{gy}^2 n_{e0} v_{e0}}{k_s} \text{ (2)}$$

where $k_s = 6.28 \times 10^{18}$ charges/second A ,

$$v_{e0}^2 = \frac{2E_0}{me} \text{ (2b)}$$

and $n_{e0} = n_e r_b$. The gyro radius is

$$r_{gy}^2 = \frac{2E}{r_e B_b^2} \text{ (3)}$$

where B_b is the net effective cusp field with maximum diamagnetic field reduction.

The magnetic field induced by this current is taken to be

$$B_{dc} = \frac{I_{de}}{2r_{gy}} = \frac{\pi r_{gy} n_{e0} v_{e0}}{2k_s} \quad (4)$$

for B_{dc} in gauss and r_{gy} in cm, or

$$B_{dc} = \frac{\pi c}{ek_s} \left[\frac{n_{e0} E_0}{B_b} \right] \quad (5)$$

Now, the net cusp field B_b is just the undisturbed central axis field B_0 less this bucking field, $B_b = B_0 - B_{dc}$. Limiting n_{e0} to the $\beta = 1$ value

$$n_{e0} = \frac{3B_b^2}{16\pi E_0} \quad (5b)$$

gives

$$B_{dc} = \left[\frac{3c}{16ek_s} \right] B_b \quad (6)$$

Note also that

$$I_{de} = \left[\frac{3}{8k_s r_c} \right] \sqrt{\frac{2E_0}{m_e}} \quad (6b)$$

at this condition, from equations (2), (3). Solving for B_b yields

$$B_b = \frac{B_0}{1 + (3c/16ek_s)} = 0.35 B_0 \quad (7)$$

For this extreme case, then, $k_L = 2.87$

However, this case ignores the intercusp field distribution which is largely undisturbed by cusp region diamagnetic currents. Thus the undisturbed surface field will remain at an amplitude B_0 at positions beyond that distance from the cusp axis at which these currents can be effective. As a result, the extent of the effective diamagnetic cusp current region will not actually scale as rapidly as r_{gy}^2 . Rather it will tend to be limited to a region of surface thick-

ness comparable to the gyro radius in the undisturbed field B_0 . This gives the effective gyro radius as the geometric mean of the diamagnetic and undisturbed field gyro radii. Under this limitation the current-induced magnetic field becomes

$$B_{dc} = \left[\frac{3c}{16ek_s} \right] \frac{B_b^2}{B_0} \quad (8)$$

so that the net effective diamagnetic field is

$$B_b = f_D B_0 = B_0 \frac{\sqrt{1+4\chi}-1}{2\chi}; \quad \chi f = \frac{3c}{16ek_s} \quad (9)$$

This gives $B_b = 0.51 B_0$ thus $k_L = 1.95$ under this condition. For purposes of calculation the loss radius factor is taken as $2 < k_L < 3$ and the effective diamagnetic factor $f_D = 1/k_L$ as $0.33 < f_D < 0.5$.

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