

# Fusion Lifetime Limits on Ion Upscattering

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The maximum degree of ion collisional upscatter allowed in a Polywell /SCIF device is that which can occur over the lifetime of ions in the system. This is limited by the ion lifetime for fusion, as well as by other loss and/or inhibiting mechanisms, if any. In the absence of other such processes, fusion is the only ion upscatter alternative. The effect of fusion lifetime on limiting upscatter is examined in this note.

The fusion rate density is just

$$q_f(r) = [n(r)]^2 \sigma(E)v(r) \quad (1b)$$

at any point in the system.

The fusion rate over the complete system volume is the integral of this over  $0 \leq r \leq R$ , thus

$$Q_f = \int_0^R [n(r)]^2 \sigma(E(r))v(r) 4\pi r^2 dr \quad (1)$$

All of the effective fusion reactions will take place where the ion density and energy are both large. This is well within the radial position  $r = 0.83R$  over which the ion density scales as the inverse square of the radius, thus the fusion rate  $Q_f$  can be evaluated without significant error by use of  $n_c(r) = n_c(r_c/r)^2$  for  $r \geq r_c$  and  $n_c(r) = n_c$  for  $0 \leq r \leq r_c$ . For simplicity take the ion energy distribution in the polyhedral field to be

$$E_i(r) = E_0 \left[ 1 - \left( \frac{r}{R} \right)^3 \right] \quad (2)$$

and the fusion cross-section to vary as

$$\sigma_f(E) = \sigma_{f0} E^3 = \sigma_{f0} \left\{ E_0 \left[ 1 - \left( \frac{r}{R} \right)^3 \right] \right\}^3 \quad (3)$$

With these the fusion rate becomes

$$Q_f = \frac{4\pi}{3} \sqrt{\frac{2}{m_i}} r_c^3 n_c^2 \sigma_{f0} E_0^{s+0.5} \frac{\left\{ 1 + 3r_c \int_r^R dr \left[ 1 - (r/R)^3 \right]^{s+0.5} \right\}}{r^2} \quad (4)$$

Assuming that the effective fusion region is within  $(r/R) \ll 1$ , equation (4) can be expanded and integrated to yield

$$Q_f = \frac{4\pi}{3} \sqrt{\frac{2}{m_i}} r_c^3 n_c^2 \sigma_{f0} E_0^{s+0.5} \left[ 4 - (3/4)(2s+5) \langle r_c \rangle \right] \quad (5)$$

where  $\langle r_c \rangle = (r_c/R)$ , and the ratio of fusions outside the core to those within it is simply

$$\frac{Q_f(out)}{Q_f(in)} = 3 - (3/4)(2s+5) \langle r_c \rangle \quad (6)$$

Dividing  $Q_f$  by the total number of ions  $N_i$  in the system gives the fusion rate per ion. Inverting this yields the fusion lifetime  $t_{fus}$  thus

$$t_{fus} = \frac{N_i}{Q_f} \quad (7)$$

Scattering collisions that tend to "thermalize" or "Maxwellianize" the ion energy distribution can continue only over a time period less than this lifetime. Beyond this time the ions are gone, and fresh, unscattered ions take their place. Thus, the fusion process provides an upper limit on the degree to which the radially-monoenergetic initial ion energy distribution can be Maxwellianized, and thus on the maximum value of the parameter  $f = \delta v/v$  to which an ion can be upscattered. This can be determined by setting the fusion lifetime  $t_{fus}$  from equations (5,7) equal to the scattering time  $t_{col}$  previously derived,<sup>1</sup> as

$$t_{col} = \frac{2(1+f)^2 f^2 E_0^{1.5} \left( \frac{v_c}{v_m} \right) \sqrt{\frac{Am_p}{2}}}{\pi^2 (Ze)^4 n_c \langle r_c \rangle} \quad (8a)$$

or

$$t_{col} = 7.7 \times 10^6 \frac{(1+f)^2 f^2 E_0^{1.5} \left( \frac{v_c}{v_m} \right) \sqrt{\frac{Am_p}{2}}}{Z^4 n_c \langle r_c \rangle} \quad (8b)$$

Equations 8a and 8b are a functionally equivalent but simpler form of the more exact expression derived by Rosenberg and Krall.<sup>2</sup>

Combining equations 5, 7, and 8, taking  $N_i = (4\pi/3)R^3 n_i$ , where  $n_i$  is the average ion density in the system, setting  $v_c = v_m$ , and simplifying the last term of equation (5) to the approximate form  $4(1-\langle r_c \rangle)$  yields an expression for the fusion-stabilized maximum upscatter as

$$(1+f)^2 f^2 = \left[ \frac{\pi^2 (Ze)^4}{8\sigma_f E_0^2} \right] \left[ \frac{(n_i/n_c)}{\langle r_c \rangle^2 (1-\langle r_c \rangle)} \right] \quad (9)$$

for cgs units, with  $E_0$  in ergs and  $\sigma_f$  in  $\text{cm}^2$ . This can be reduced further by use of equation (3) for the fusion cross-section, as desired. From this equation it is evident that

$$[f \langle r_c \rangle]^2 \approx \frac{(n_o/n_c)}{E_0^{s+2}} \quad (9b)$$

for the usual cases where  $f, \langle r_c \rangle \ll 1$ .

The determination of  $n_i$  and/or  $n_i/n_c$  can be made exactly by direct computation within the EKXL code. This is the method followed for use of this equation in the code as an upper limit on velocity upscatter. Here, however, it is useful to employ the analytic forms for  $(n_i/n_c)$  derived by Krall<sup>3</sup> for a specified potential distribution. Taking the forms he found for ion density variation in each of the (three) regions of the system, and integrating these over these regions yields the approximate result

$$n_i \approx \frac{3}{4} (1 + 2\langle r_c \rangle) \langle r_c \rangle n_R \quad (10)$$

where  $n_R = n_i(R)$ . Note that this is less than the normalizing density used by Krall at the point  $r = R/2$ , which he gives as  $n_i(r/2) = n_o = 3n_R \langle r_c \rangle$ . In this work<sup>3</sup> it is also shown that  $n_c = (3/2)n_R \langle r_c \rangle$ , so that the density ratio  $(n_i/n_c) = (1/2)(1+2\langle r_c \rangle) \langle r_c \rangle^{-2}$ . Using this reduces equation (9) to

$$(1+f^2) f^2 \approx \left[ \frac{\pi^2 (Ze)^4}{8\sigma_f E_0^2} \right] \left[ \frac{1+2\langle r_c \rangle}{2(1-\langle r_c \rangle)} \right] \quad (11)$$

which is virtually independent of  $\langle r_c \rangle$  as well as of all other system design parameters, except only the maxi-

mum well depth or electron injection energy,  $E_0$ . Taking  $\langle r_c \rangle \ll 1$  and reducing this numerically gives

$$(1+f)^2 f^2 \approx \frac{1.29 \times 10^{10}}{\sigma_{fb} E_0^2} \quad (12)$$

for  $\sigma_{fb}$  in barns and  $E_0$  in eV. For example, for DD as the fuel with  $E_0 = 1 \times 10^5$  eV and  $\sigma_{fb} = 0.1b$ , then  $f = 1.45$ , while for  $E_0 = 3 \times 10^5$  eV,  $\sigma_{fb} = 0.17b$  and  $f = 0.56$ . If DT is chosen as fuel,  $E_0 = 3 \times 10^4$  eV,  $\sigma_{fb} = 4.0b$ , the upscatter is fusion-rate-limited to  $f = 0.93$ .

This shows the important result that fusion reaction rates can limit the upscatter to values such that reasonable system operating conditions (well depths) can hold the ions within the system even if no other f-stabilizing mechanisms are available.

However, other mechanisms *are* available to provide a limit on the maximum value of  $f$ . These are the isotropizing collisional effects which will occur in scatterings that take place in both the outer edge region<sup>4</sup> of the system and in the central ion core.<sup>5</sup> The outer edge isoscattering will act to transform transverse momentum into radial momentum within a limiting value of  $f$  set by a balance between core Maxwellianization upscattering time and edge energy exchange collision time. The core isoscattering will transform transverse into radial momentum at the same rate as core collisional upscattering, if the energy distribution function of the core ions rises with increasing transverse energy. This is the case for the SCIF ECRH-driven ion source, but is not the case for the source distribution used in the EKXL code; this is a square function for which no net momentum transformation can occur in core scatterings.

Isosscatterings that transform transverse into radial momentum of the ions will limit the opposite effect driven by core Maxwellian upscattering to higher energy well radii, and from isosscatterings in the predominantly radial now in the mantle region (i.e. that region between the core and the edge). These competing effects will always act to stabilize the maximum value of upscatter ( $f$ ). They can be made to provide more restrictive limits on maximum  $f$  than that of the fusion-rate-limit discussed above, by appropriate choice of system design and operating conditions. A more detailed analysis and discussion of these phenomena is given in a separate technical report.

## Publishing History

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## References

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- <sup>2</sup> Marlene Rosenberg (EMC2) and Nichols A. Krall; “Ion Loss by Collisional Upscattering”. Krall Associates technical report, KA-90-39, October 1990
- <sup>3</sup> Nicholas A. Krall; “Density Profiles in the Polywell/ SCIF Device,” Krall Associates technical report KA-90-51, December 1990
- <sup>4</sup> Marlene Rosenberg (EMC2) and Nicholas A. Krall; “Ion Loss by Collisions Outside the Core,” Krall Associates technical report, KA-91-02, January 1991
- <sup>5</sup> Robert W. Bussard, G.P. Jellison, and G.E. McClellan; “Preliminary Research Studies of a New Method for Control of Charged Particle Interactions”, PSR Report 1899, Pacific-Sierra Research Corp., Final Report under DNA Contract No. DNA001-87-C-0052, November 30, 1988. Appendix A. Section on “Ion Convergence, Collisional Cores, and Integration Limits.” pp. 103-105